Relative Velocity, Vectors, and Motion Graphs Worksheet

Burke Tutoring In Fremont · 27 problems in five parts · answer key at the end

Name _____ Class _____ Date _____

Directions. Draw a sketch for every problem: a vector triangle for Parts A to C and labeled axes for Part D. Take east and north as positive unless the problem says otherwise. Bearings are measured clockwise from north.

Round final answers to three significant figures and angles to the nearest tenth of a degree. Keep unrounded values between steps.

Guide, simulators, and worked examples: burketutoringinfremont.com/relative-velocity-vectors-motion-graphs/

Part A: Vectors and components

Components, directions, and adding vectors.

1. A displacement of 12.0 m points 35.0° north of east. Find its east and north components.

2. A velocity has components $v_x = -6.0$ m/s and $v_y = +8.0$ m/s, with east and north positive. Find its magnitude and direction.

3. A truck drives at 50 km/h on a bearing of 240°. Write this direction as an angle south of west, then find the east and north components of the velocity.

4. A hiker walks 3.0 km due north, then 5.0 km in the direction 30.0° south of east. Find the magnitude and direction of her total displacement.

5. A car's velocity changes from 20 m/s due north to 20 m/s due east in 4.0 s. Its speed never changes. Find the change in velocity and the average acceleration, with directions.

Part B: Relative velocity in one dimension and river crossings

Problems 6 and 7 are one-dimensional, the form AP Physics 1 tests. Problems 8 to 11 cross the river.

6. A moving walkway 60 m long moves at 1.2 m/s. A traveler walks at 1.5 m/s relative to the walkway. How long does the trip take if she walks in the direction the walkway moves? How long if she walks the full length against it?

7. A boat travels 12 km downstream in 0.75 h, then returns the same 12 km upstream in 1.5 h. Its speed relative to the water is the same both ways. Find the boat's speed relative to the water and the speed of the current.

8. A river 80 m wide flows east at 1.5 m/s. A swimmer swims at 2.0 m/s relative to the water and aims straight across, due north. Find the crossing time, how far downstream she lands, and her speed relative to the ground.

9. The swimmer from problem 8 wants to land directly opposite her starting point. In what direction must she aim, and how long does the crossing take?

10. A motorboat crosses a river 120 m wide that flows east at 3.0 m/s. The boat moves at 5.0 m/s relative to the water, heading 20.0° west of north. Find the crossing time and where the boat lands relative to the point directly opposite.

11. A canoeist paddles at 1.0 m/s relative to the water in a river that flows at 2.0 m/s. Can she land directly opposite her starting point? Explain using components. If she aims straight across a river 50 m wide, how far downstream does she land?

Part C: Planes, wind, and moving observers

Bearings are measured clockwise from north. Problem 12 is one-dimensional.

12. A plane with an airspeed of 180 km/h flies 360 km due east, then 360 km due west back to its starting airport. A 40 km/h wind blows toward the east the whole time. Find the time for each leg and for the whole trip. How does the total compare with the same trip in still air?

13. A plane heads due north with an airspeed of 240 km/h. A 70 km/h wind blows toward the east. Find the plane's ground speed and the direction it actually travels.

14. A pilot wants to travel due north to a city 450 km away. The plane's airspeed is 300 km/h, and a 60 km/h wind blows from the west. What heading should she fly, what is her ground speed, and how long does the trip take?

15. A plane flies with a heading of 090° (due east) and an airspeed of 200 km/h. The wind blows toward 045° (northeast) at 50.0 km/h. Find the ground speed and the track.

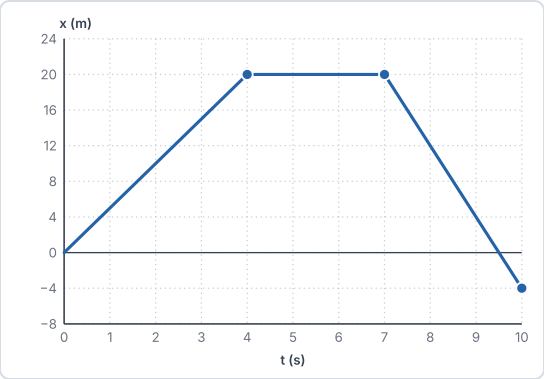
16. A pilot wants a track due east (090°). The airspeed is 220 km/h, and the wind blows toward 135° (southeast) at 40.0 km/h. Find the heading and the ground speed.

17. Rain falls straight down at 8.0 m/s relative to the ground. A car drives east at 15 m/s. Find the speed of the raindrops relative to the car and the angle their paths make with the vertical, as seen by the driver.

Part D: Motion graphs

Straight-line motion with constant acceleration in each stage.

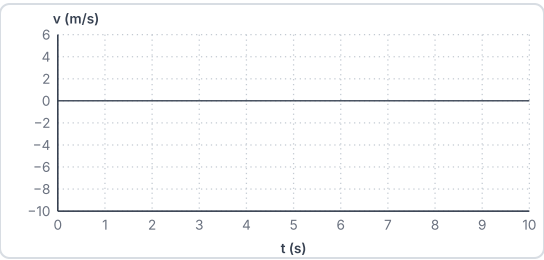
18. The graph shows a cyclist's position on a straight road, with east positive. (a) Find the velocity in each of the three stages. (b) Sketch the velocity versus time graph. (c) Find the displacement, the distance, the average velocity, and the average speed for the full 10 s.



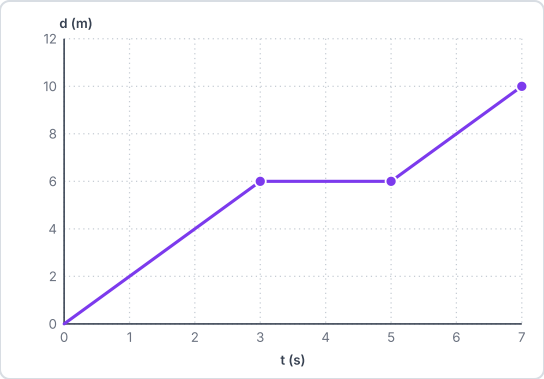
KEY

———	Position of the cyclist, east positive
●	Corners at (4 s, 20 m), (7 s, 20 m), (10 s, −4 m)

Sketch here



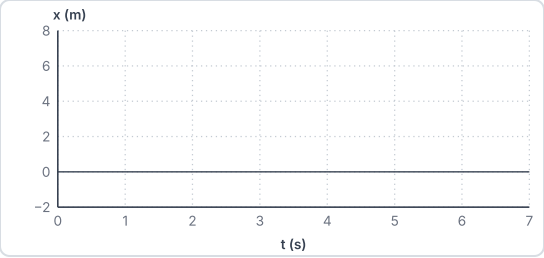
19. The graph shows the distance a walker has traveled. She starts at x = 0 and walks east first. In the last stage, from 5 s to 7 s, she walks west. (a) Sketch her position versus time graph, with east positive. (b) Find her final position. (c) Explain why a distance versus time graph can never slope downward.



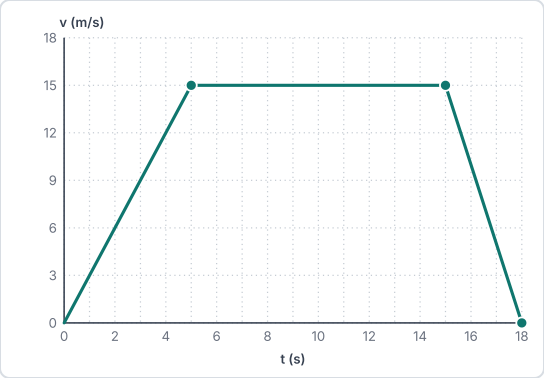
KEY

———	Distance traveled by the walker
●	Corners at (3 s, 6 m), (5 s, 6 m), (7 s, 10 m)

Sketch here



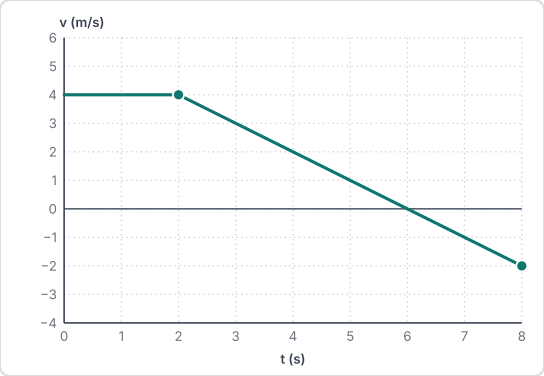
20. A car pulls away from a stop light, cruises, and stops at the next light. (a) Find the acceleration in each stage. (b) Find the distance between the lights. (c) Find the average velocity for the trip. (d) At what time has the car covered half the distance?



KEY

—	Velocity of the car
●	Corners at (5 s, 15 m/s), (15 s, 15 m/s), (18 s, 0)

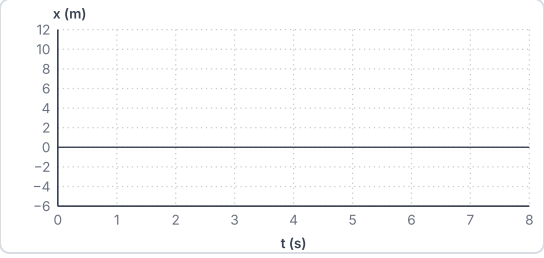
21. A cart on a track starts at x = −5.0 m and has the velocity graph shown. (a) Find the cart's position at t = 2.0 s and at t = 8.0 s. (b) When does it turn around, and where? (c) Find the total distance traveled from 0 to 8.0 s. (d) At what two times is the cart at x = 10 m?



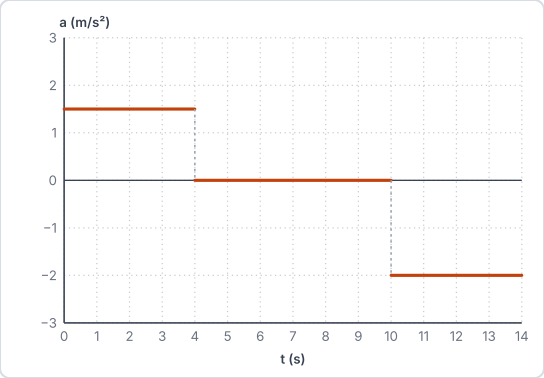
KEY

—	Velocity of the cart
●	Corners at (2 s, 4 m/s) and (8 s, −2 m/s)

Sketch here



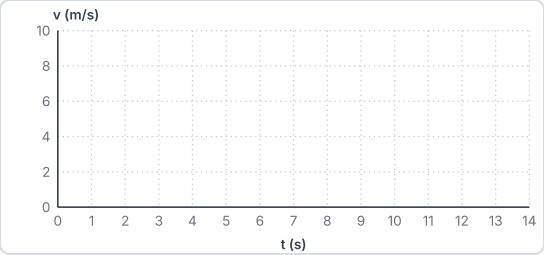
22. A train moving at +2.0 m/s at t = 0 has the acceleration shown. (a) Find the velocity at 4.0 s, 10 s, and 14 s. (b) Sketch the velocity versus time graph. (c) Find the displacement over the 14 s.



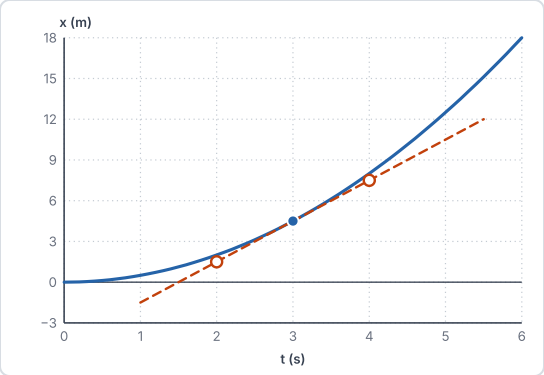
KEY

▬	Acceleration of the train: 1.5, 0, and −2.0 m/s² in turn
┄	Jumps between stages

Sketch here



23. The curve shows the position of a cart that starts from rest at t = 0 and has constant acceleration. A tangent line is drawn at t = 3.0 s. (a) Find the instantaneous velocity at 3.0 s. (b) Find the average velocity from 0 to 3.0 s, and explain why it is smaller. (c) Find the acceleration.

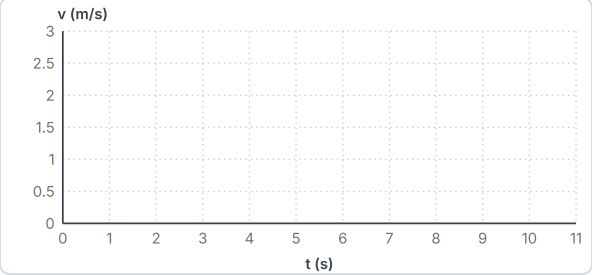


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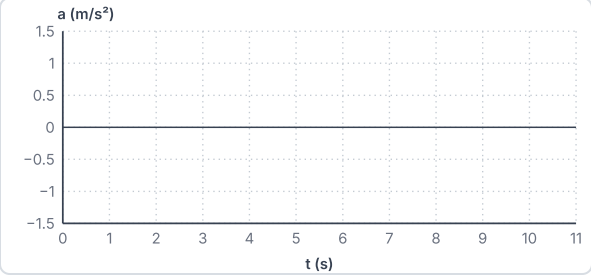
▬	Position of the cart
┄	Tangent line at t = 3.0 s
●	Point of tangency, (3 s, 4.5 m)
○	Two points on the tangent line: (2 s, 1.5 m) and (4 s, 7.5 m)

24. An elevator starts at rest on the ground floor and rises. It speeds up uniformly to 2.0 m/s in 2.0 s, moves at constant speed for 6.0 s, then slows uniformly to a stop in 2.5 s. Take up as positive. (a) Sketch the velocity versus time and acceleration versus time graphs. (b) Find the acceleration in each stage. (c) How high does the elevator rise?

Sketch here



Sketch here



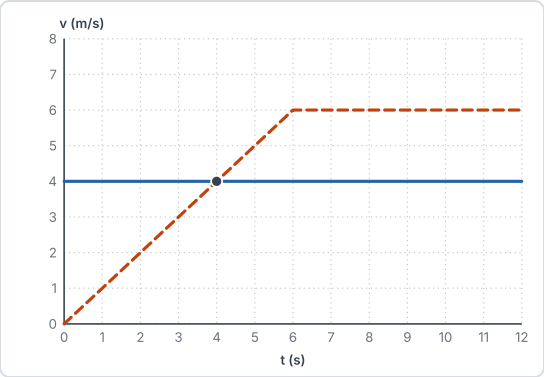
Challenge set: problems 25 to 27

Longer problems that combine the ideas above.

25. A boat moves at 3.0 m/s relative to the water in a river 60 m wide that flows at 5.0 m/s. (a) Show that the downstream drift is smallest when the boat aims upstream at an angle θ from straight across with sin θ = 3.0 / 5.0. (b) Find that drift and the crossing time. (c) Compare with aiming straight across.

26. A plane with an airspeed of 200 km/h flies 300 km due north and returns. Compare the round-trip time in three cases: still air, a 50 km/h wind blowing toward the east (across the route), and a 50 km/h wind blowing toward the north (along the route).

27. Carts P and Q pass the same point at t = 0, moving in the same direction. Their velocity graphs are shown. (a) At t = 4.0 s, where the lines cross, which cart is ahead and by how much? (b) When and where does Q catch P? (c) Explain why the crossing point on the velocity graph is not where the carts meet.



KEY

———	Cart P
– – –	Cart Q
●	The velocity lines cross at (4 s, 4 m/s)

Answer key

Worked answers for all 27 problems.

Part A: Vectors and components

1. East: 12.0 cos 35.0° = 9.83 m. North: 12.0 sin 35.0° = 6.88 m.

2. Magnitude $\sqrt{(6.0^2 + 8.0^2)} = 10.0$ m/s. $\tan^{-1}(8.0 / 6.0) = 53.1°$, and the vector points west and north, so the direction is 53.1° north of west. The same direction can be written 36.9° west of north, or a bearing of 323.1°.

3. A bearing of 240° is 60° clockwise past due south, which is 30.0° south of west. East: 50 sin 240° = −43.3 km/h. North: 50 cos 240° = −25.0 km/h. Both are negative because the truck moves west and south.

4. Second leg: (5.0 cos 30.0°, −5.0 sin 30.0°) = (4.33, −2.50) km. Total: (4.33, 3.0 − 2.50) = (4.33, 0.50) km. Magnitude $\sqrt{(4.33^2 + 0.50^2)} = 4.36$ km. Direction $\tan^{-1}(0.50 / 4.33) = 6.6°$ north of east.

5. $\Delta v = v_{final} - v_{initial} = (20, 0) - (0, 20) = (20, -20)$ m/s, with magnitude 28.3 m/s pointing 45.0° south of east. Average acceleration = $\Delta v / \Delta t = 7.07$ m/s², also 45.0° south of east. The car accelerates even though its speed is constant, because the direction of its velocity changes.

Part B: Relative velocity in one dimension and river crossings

6. With the walkway: 1.5 + 1.2 = 2.7 m/s, so t = 60 / 2.7 = 22.2 s. Against it: 1.5 − 1.2 = 0.30 m/s, so t = 60 / 0.30 = 200 s.

7. Downstream: 12 / 0.75 = 16 km/h = $v_b + v_c$. Upstream: 12 / 1.5 = 8.0 km/h = $v_b - v_c$. Adding the equations gives $2v_b = 24$, so $v_b = 12$ km/h and $v_c = 4.0$ km/h.

8. t = 80 / 2.0 = 40.0 s. Drift = (1.5)(40.0) = 60.0 m downstream. Ground speed $\sqrt{(2.0^2 + 1.5^2)} = 2.50$ m/s, in the direction 36.9° east of north.

9. Her upstream component must cancel the current: 2.0 sin θ = 1.5, so θ = 48.6° upstream of straight across, which is 48.6° west of north. Across component: $\sqrt{(2.0^2 - 1.5^2)} = 1.32$ m/s. Time: 80 / 1.3229 = 60.5 s.

10. Across: 5.0 cos 20.0° = 4.70 m/s, so t = 120 / 4.698 = 25.5 s. Along the river: 3.0 − 5.0 sin 20.0° = 3.0 − 1.71 = 1.29 m/s downstream. Drift = (1.290)(25.54) = 32.9 m downstream of the point directly opposite.

11. No. The upstream component of her velocity relative to the water is at most 1.0 m/s, which cannot cancel a 2.0 m/s current, so every heading carries her downstream. Aiming straight across: t = 50 / 1.0 = 50 s, and drift = (2.0)(50) = 100 m downstream.

Part C: Planes, wind, and moving observers

12. East leg: ground speed 180 + 40 = 220 km/h, t = 360 / 220 = 1.64 h. West leg: 180 − 40 = 140 km/h, t = 360 / 140 = 2.57 h. Total: 4.21 h, compared with 720 / 180 = 4.00 h in still air. The headwind leg costs more time than the tailwind leg saves.

13. Ground speed $\sqrt{(240^2 + 70^2)} = 250$ km/h. Direction $\tan^{-1}(70 / 240) = 16.3°$ east of north, a track of 016.3°.

14. A wind from the west blows toward the east, so she aims west of north: 300 sin θ = 60, θ = 11.5° west of north, a heading of 348.5°. Ground speed $\sqrt{(300^2 - 60^2)} = 294$ km/h. Time 450 / 293.9 = 1.53 h, about 92 minutes.

15. Wind components: (50.0 sin 45°, 50.0 cos 45°) = (35.4, 35.4) km/h. Ground velocity: (200 + 35.4, 35.4) = (235.4, 35.4) km/h. Ground speed 238 km/h. Track: $\tan^{-1}(35.4 / 235.4) = 8.5°$ north of east, a bearing of 081.5°.

16. Wind components: (40.0 sin 135°, 40.0 cos 135°) = (28.3, −28.3) km/h, so the wind pushes the plane 28.3 km/h south. The plane must supply 28.3 km/h north: 220 sin α = 28.3, α = 7.4° north of east, a heading of 082.6°. Ground speed: 220 cos 7.39° + 28.3 = 218.2 + 28.3 = 246 km/h.

17. $v_{rain,\,car} = v_{rain,\,ground} - v_{car,\,ground} = (0, -8.0) - (15, 0) = (-15, -8.0)$ m/s. Speed $\sqrt{(15^2 + 8.0^2)} = 17.0$ m/s. Angle from vertical: $\tan^{-1}(15 / 8.0) = 61.9°$. The drops move toward the back of the car as they fall, so the driver sees them coming from ahead.

Part D: Motion graphs

18. (a) 0 to 4 s: 20 / 4 = +5.0 m/s. 4 to 7 s: 0. 7 to 10 s: (−4 − 20) / 3 = −8.0 m/s. (b) Three horizontal segments at +5.0, 0, and −8.0 m/s, with jumps at 4 s and 7 s. (c) Displacement −4.0 m, which is 4.0 m west of the start. Distance 20 + 24 = 44 m. Average velocity −4.0 / 10 = −0.40 m/s. Average speed 44 / 10 = 4.4 m/s.

19. (a) x rises from 0 to 6 m over 0 to 3 s (slope +2.0 m/s), stays at 6 m from 3 to 5 s, then falls to 2 m at 7 s (slope −2.0 m/s). (b) x = +2.0 m. (c) Distance adds the length of every bit of path traveled, so it can increase or stay the same but cannot decrease.

20. (a) 15 / 5 = +3.0 m/s²; 0; −15 / 3 = −5.0 m/s². (b) Area: ½(5)(15) + (10)(15) + ½(3)(15) = 37.5 + 150 + 22.5 = 210 m. (c) 210 / 18 = 11.7 m/s. (d) Half is 105 m. The first stage covers 37.5 m, leaving 67.5 m at 15 m/s, which takes 4.5 s: t = 9.5 s.

21. (a) x(2) = −5.0 + (4)(2) = 3.0 m. From 2 to 8 s the area is ½(4)(4) − ½(2)(2) = 8 − 2 = +6 m, so x(8) = 9.0 m. (b) v = 0 at t = 6.0 s, where x = 3.0 + 8.0 = 11 m. (c) 8 + 8 + 2 = 18 m. (d) For t ≥ 2 s, with u = t − 2: x = 3 + 4u − 0.5u². Setting x = 10 gives u² − 8u + 14 = 0, so u = 4 ± √2. The cart passes x = 10 m at t = 4.59 s moving forward and at t = 7.41 s on the way back.

22. (a) v(4) = 2.0 + (1.5)(4) = 8.0 m/s. v(10) = 8.0 m/s. v(14) = 8.0 − (2.0)(4) = 0. (b) A line rising from 2.0 to 8.0 m/s over 0 to 4 s, level at 8.0 m/s until 10 s, then falling to 0 at 14 s. (c) ½(2 + 8)(4) + (8)(6) + ½(8)(4) = 20 + 48 + 16 = 84 m.

23. (a) Slope of the tangent: (7.5 − 1.5) / (4 − 2) = 3.0 m/s. (b) 4.5 / 3.0 = 1.5 m/s. The cart speeds up the whole time, so its average velocity is less than its velocity at the end of the interval; for constant acceleration from rest it is exactly half. (c) a = (3.0 − 0) / 3.0 = 1.0 m/s².

24. (a) Velocity: a line from 0 to 2.0 m/s over 0 to 2 s, level until 8 s, then down to 0 at 10.5 s. Acceleration: +1.0 m/s² from 0 to 2 s, 0 from 2 to 8 s, −0.80 m/s² from 8 to 10.5 s. (b) +1.0, 0, and −0.80 m/s². (c) ½(2)(2) + (2)(6) + ½(2)(2.5) = 2 + 12 + 2.5 = 16.5 m.

Challenge set: problems 25 to 27

25. (a) Across: 3.0 cos θ. Along: 5.0 − 3.0 sin θ. Drift = 60(5.0 − 3.0 sin θ) / (3.0 cos θ). Setting its derivative to zero gives 5.0 sin θ − 3.0 = 0, so sin θ = 0.60 and θ = 36.9°. Geometrically, the ground velocity is then perpendicular to the boat's velocity relative to the water. (b) Across: (3.0)(0.80) = 2.4 m/s, so t = 60 / 2.4 = 25 s. Along: 5.0 − 1.8 = 3.2 m/s, so drift = (3.2)(25) = 80 m. (c) Straight across: t = 20 s and drift = 100 m. The best heading saves 20 m of drift and costs 5 s.

26. Still air: 600 / 200 = 3.00 h. Crosswind: each leg needs a ground speed of √(200² − 50²) = 193.6 km/h, so 600 / 193.6 = 3.10 h. Along the route: 300 / 250 + 300 / 150 = 1.20 + 2.00 = 3.20 h. The wind lengthens the round trip in both cases, and the wind along the route costs more time.

27. (a) Areas from 0 to 4 s: P covers (4)(4) = 16 m and Q covers ½(4)(4) = 8.0 m, so P leads by 8.0 m. This is the largest gap, because Q is faster from then on. (b) At 6 s, Q has covered ½(6)(6) = 18 m and P has covered 24 m. After that, $x_Q = 18 + 6(t − 6)$ and $x_P = 4t$, which are equal at t = 9.0 s and x = 36 m. (c) Equal velocities mean the gap stops growing at that instant. The carts meet when the areas under their velocity graphs are equal.

Free to print and use in class. Interactive river, plane, and graph tools for these problems: burketutoringinfremont.com/relative-velocity-vectors-motion-graphs/