Vectors, Relative Velocity, and Motion Graphs

Reference sheet · kinematics · east and north positive · bearings clockwise from north

Burke Tutoring In Fremont
burketutoringinfremont.com
(510) 453-0350

COMPONENTS AND DIRECTIONS

Angle θ from east	$x = v\cos\theta$, $y = v\sin\theta$
Bearing β, clockwise from north	east = $v\sin\beta$, north = $v\cos\beta$
From components	$v = \sqrt{(v_x^2 + v_y^2)}$, $\tan\theta = \lvert v_y\rvert / \lvert v_x\rvert$

30° north of east = N 60° E = bearing 060°. Name the quadrant from the signs of the components.

RELATIVE VELOCITY

$\mathbf{v_{AC} = v_{AB} + v_{BC}}$, and $v_{BA} = -v_{AB}$.

- Boat: $v_{BG} = v_{BW} + v_{WG}$. Plane: $v_{PG} = v_{PA} + v_{AG}$.
- One dimension: downstream $v_b + v_c$, upstream $v_b - v_c$; tailwind $v_a + w$, headwind $v_a - w$.
- AP Physics 1 limits relative velocity addition to one dimension.

RIVER CROSSINGS

Current v_c along +x, width W, boat speed v_b aimed at θ upstream of straight across.

Across: $v_b\cos\theta$. Along: $v_c - v_b\sin\theta$. Time $t = W / (\text{across})$. Drift = (along) × t.

Goal	Heading	Time
Least time	Straight across	W / v_b; drift v_cW / v_b
Land directly opposite ($v_b > v_c$)	$\sin\theta = v_c / v_b$	$W / \sqrt{(v_b^2 - v_c^2)}$
Least drift ($v_c > v_b$)	$\sin\theta = v_b / v_c$	$W / (v_b\cos\theta)$

PLANES AND WIND

- Heading: where the nose points. Track: where the plane goes.
- A wind from the west blows toward the east.
- Hold a course: $v_a\sin(\text{WCA}) = w_{across}$, then ground speed = $v_a\cos(\text{WCA}) + w_{along}$.
- Heading given: add the components of v_{PA} and the wind.

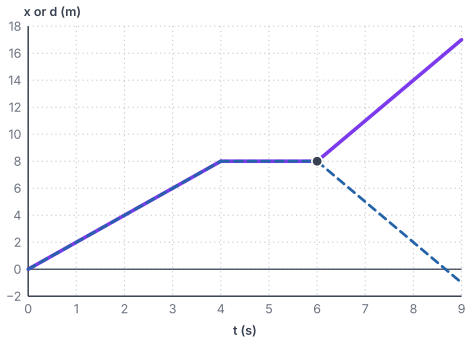
READING MOTION GRAPHS

- Slope of x: velocity. Slope of d: speed. Slope of v: acceleration.
- Signed area under v: Δx. Area under a: Δv.
- d never decreases. It matches $x - x_0$ until the first turnaround if the motion starts in the + direction.
- v crosses zero where x has a peak or valley.
- Speeding up when v and a share a sign.

FIVE CHECKS BEFORE YOU HAND IT IN

1. Does each wind or current arrow point the way the air or water moves?
2. Did I use the across component, not the full speed, for a crossing time?
3. Is the answer a heading, a track, or a ground speed, and is that what the question asked?
4. Which graph am I reading, and does the question want its slope, its area, or its value?
5. Is the size sensible? A sum of two vectors lies between the difference and the sum of their sizes.

POSITION VERSUS DISTANCE, SAME WALK



– – –	Position x (m), east positive
———	Distance d (m)
●	Turnaround at t = 6.0 s: the two graphs separate here

Free to print and use in class. Guide, simulators, and 27 practice problems: burketutoringinfremont.com/relative-velocity-vectors-motion-graphs/