Rational Equations Worksheet

with excluded values, extraneous solution checks, and a full answer key

Name _____ Date _____ Period _____

The two list method

List 1, before you solve. Factor every denominator and write down every value that makes a factor zero. These are the excluded values.

List 2, after you solve. Multiply *every* term by the LCD, solve the equation that is left, and write down the roots. These are candidates.

Compare. Cross out any candidate that appears on List 1. Clearing denominators only produces an equivalent equation for values the original equation allows, so the polynomial can carry roots the original does not have. Substitute the survivors into the *original* equation, not the cleared one, then state the solution set, which is allowed to be empty.

Word problems can also produce a candidate that satisfies the equation but makes no sense in the situation, such as a negative time. That value is non viable rather than extraneous. Say which reason you are using.

Warm up Excluded values only

Do not solve these. List every value that would make a denominator zero.

1. $\frac{3}{x} = \frac{5}{x+2}$ _____

2. $\frac{1}{x-7} + 2 = \frac{4}{x-7}$ _____

3. $\frac{x}{x^2-16} = \frac{1}{x+4}$ _____

4. $\frac{5}{2x-6} = \frac{1}{x}$ _____

5. $\frac{1}{x^2+7x} = \frac{3}{x+7}$ _____

6. $\frac{4}{x^2-5x+6} = \frac{1}{x-2}$ _____

7. $\frac{x}{3x+5} = \frac{2}{x-1}$ _____

8. $\frac{2}{x^2-1} + \frac{1}{x+1} = 5$ _____

9. $\frac{6}{x^2+2x-15} = \frac{1}{x-3}$ _____

10. $\frac{1}{4x} - \frac{1}{x^2} = 2$ _____

Scratch Factoring space for the warm up

Show the factored form of any denominator that needed one.

Part A Proportions: one fraction on each side

Cross multiplication works on every problem here. Write the excluded values anyway.

1. $\frac{5}{x+2} = \frac{3}{x-4}$

2. $\frac{6}{x-1} = \frac{2}{x+5}$

3. $\frac{4}{x} = \frac{6}{x+5}$

4. $\frac{x-2}{x+3} = \frac{3}{5}$

5. $\frac{x}{x+4} = \frac{3}{x+2}$

6. $\frac{x+1}{x-5} = \frac{6}{x-5}$

Part B One LCD, linear after clearing

Multiply every term, including the whole numbers. One of these six has no solution.

7. $\frac{3}{x} + \frac{1}{2} = \frac{5}{x}$

8. $\frac{x}{x - 2} - 3 = \frac{4}{x - 2}$

9. $\frac{x}{x + 3} + 1 = \frac{7}{x + 3}$

10. $\frac{2}{x - 4} = \frac{3}{x - 4} + 1$

11. $\frac{x}{x + 5} - 2 = -\frac{5}{x + 5}$

12. $\frac{2}{x - 1} - \frac{1}{x + 1} = 0$

Part C Factorable quadratic denominators, problems 13 to 16

Factor first. The factored denominator is where the excluded values are hiding.

13. $\frac{1}{x - 2} + \frac{1}{x + 2} = \frac{4}{x^2 - 4}$

14. $\frac{4}{x - 2} - \frac{3}{x + 2} = \frac{14}{x^2 - 4}$

15. $\frac{1}{x} + \frac{1}{x - 2} = \frac{8}{x^2 - 2x}$

16. $\frac{x^2}{x - 3} = \frac{9}{x - 3}$

Part C Factorable quadratic denominators, problems 17 to 19

In problem 19 a factor cancels, so the excluded value is a hole in the graph rather than an asymptote. Take the excluded values from the denominator as it was written, before you cancel anything.

17. $\dfrac{x}{x+2} - \dfrac{1}{x-2} = \dfrac{8}{x^2-4}$

18. $1 - \dfrac{3}{x} = \dfrac{10}{x^2}$

19. $\dfrac{x^2-4}{x-2} = 3x - 4$

Part D Mixed and applied, problems 20 to 22

Define your variable, write the equation, then use the same routine. State units in the answer.

20. Machine A fills a hopper in 6 hours. Running with machine B the two of them fill it in 4 hours. How long does machine B take on its own?

21. Hose B takes 5 hours longer to fill a pool than hose A. Together they fill it in 6 hours. How long does each hose take alone?

22. A boat travels 24 miles downstream in the same time it travels 16 miles upstream. In still water the boat does 10 miles per hour. How fast is the current?

Part D Mixed and applied, problems 23 to 25

One of these is not a normal equation. Read the result carefully.

23. The sum of a number and its reciprocal is 10/3. Find every such number.

24. $\dfrac{2x}{x^2 - 1} = \dfrac{1}{x - 1} + \dfrac{1}{x + 1}$

25. A student has 12 correct answers out of 20 on a practice set. Every new answer she gives from now on is correct. How many more questions does she need to answer to reach 80 percent?

Challenge Problems 26 to 28

Same routine, heavier algebra. One of the three has no solution.

26. $\frac{x+1}{x-1} - \frac{x-1}{x+1} = \frac{8}{x^2-1}$

27. $\frac{1}{x-2} + \frac{1}{x-3} = -\frac{1}{x^2-5x+6}$

28. $\frac{x}{x-2} - \frac{4}{x+2} = \frac{16}{x^2-4}$

Answer key

Excluded values, the cleared equation, the candidates, and the verdict on each one.

Warm up. Excluded values only.

1. $x = 0$, $x = -2$

2. $x = 7$

3. $x = 4$, $x = -4$

4. $x = 3$, $x = 0$

5. $x = 0$, $x = -7$

6. $x = 2$, $x = 3$

7. $x = -5/3$, $x = 1$

8. $x = 1$, $x = -1$

9. $x = 3$, $x = -5$

10. $x = 0$

#	EXCLUDED	CLEARED EQUATION AND CANDIDATES	ANSWER
1	-2, 4	$5(x - 4) = 3(x + 2)$, so $2x = 26$ Candidates: 13	$x = 13$
2	1, -5	$6(x + 5) = 2(x - 1)$, so $4x = -32$ Candidates: -8	$x = -8$
3	0, -5	$4(x + 5) = 6x$, so $2x = 20$ Candidates: 10	$x = 10$
4	-3	$5(x - 2) = 3(x + 3)$, so $2x = 19$ Candidates: 19/2	$x = 19/2$
5	-4, -2	$x(x + 2) = 3(x + 4)$, so $(x - 4)(x + 3) = 0$ Candidates: 4 and -3	$x = 4$ or $x = -3$
6	5	$x + 1 = 6$, so $x = 5$ Candidates: 5	No solution. The only candidate is excluded.
7	0	LCD $2x$. $6 + x = 10$ Candidates: 4	$x = 4$
8	2	LCD $x - 2$. $x - 3(x - 2) = 4$, so $-2x = -2$ Candidates: 1	$x = 1$
9	-3	LCD $x + 3$. $x + (x + 3) = 7$, so $2x = 4$ Candidates: 2	$x = 2$
10	4	LCD $x - 4$. $2 = 3 + (x - 4)$ Candidates: 3	$x = 3$
11	-5	LCD $x + 5$. $x - 2(x + 5) = -5$, so $-x = 5$ Candidates: -5	No solution. The only candidate is excluded.
12	1, -1	LCD $(x - 1)(x + 1)$. $2(x + 1) - (x - 1) = 0$ Candidates: -3	$x = -3$
13	2, -2	LCD $(x - 2)(x + 2)$. $(x + 2) + (x - 2) = 4$ Candidates: 2	No solution. The only candidate is excluded.
14	2, -2	LCD $(x - 2)(x + 2)$. $4(x + 2) - 3(x - 2) = 14$ Candidates: 0	$x = 0$. Zero is not excluded here.
15	0, 2	$x^2 - 2x = x(x - 2)$. $(x - 2) + x = 8$ Candidates: 5	$x = 5$
16	3	LCD $x - 3$. $x^2 = 9$, so $(x - 3)(x + 3) = 0$ Candidates: 3 and -3	$x = -3$. The candidate 3 is excluded.
17	2, -2	$x(x - 2) - (x + 2) = 8$, so $(x - 5)(x + 2) = 0$ Candidates: 5 and -2	$x = 5$. The candidate -2 is excluded.
18	0	LCD x^2. $x^2 - 3x = 10$, so $(x - 5)(x + 2) = 0$ Candidates: 5 and -2	$x = 5$ or $x = -2$

#	EXCLUDED	CLEARED EQUATION AND CANDIDATES	ANSWER
19	2	LCD $x - 2$. $x^2 - 4 = (3x - 4)(x - 2)$, so $2x^2 - 10x + 12 = 0$ and $2(x - 3)(x - 2) = 0$ Candidates: 3 and 2	$x = 3$. The candidate 2 is excluded. Note that $x - 2$ cancels, so the left side equals $x + 2$ everywhere except at $x = 2$. That excluded value is a hole in the graph rather than an asymptote, and it is rejected all the same.
20	$t = 0$	$1/6 + 1/t = 1/4$. LCD $12t$ gives $2t + 12 = 3t$ Candidates: 12	Machine B takes 12 hours.
21	$t = 0$, $t = -5$	$1/t + 1/(t + 5) = 1/6$. LCD $6t(t + 5)$ gives $t^2 - 7t - 30 = 0$ Candidates: 10 and -3	Hose A takes 10 hours, hose B takes 15 hours. The candidate -3 is not excluded, so it is not extraneous. It is discarded as non viable, because a hose cannot take a negative number of hours.
22	$c = 10$, $c = -10$	$24/(10 + c) = 16/(10 - c)$. Cross multiply: $24(10 - c) = 16(10 + c)$, so $40c = 80$ Candidates: 2	The current is 2 miles per hour.
23	0	$x + 1/x = 10/3$. LCD $3x$ gives $3x^2 - 10x + 3 = 0$, so $(x - 3)(3x - 1) = 0$ Candidates: 3 and 1/3	$x = 3$ or $x = 1/3$
24	1, -1	LCD $(x - 1)(x + 1)$. $2x = (x + 1) + (x - 1)$, so $2x = 2x$ Candidates: every value	An identity. Every real number works except $x = 1$ and $x = -1$.
25	$n = -20$	$(12 + n)/(20 + n) = 4/5$. $5(12 + n) = 4(20 + n)$ Candidates: 20	20 more questions.
26	1, -1	LCD $(x^2 - 1)$. $(x + 1)^2 - (x - 1)^2 = 8$, so $4x = 8$ Candidates: 2	$x = 2$
27	2, 3	LCD $(x - 2)(x - 3)$. $(x - 3) + (x - 2) = -1$, so $2x = 4$ Candidates: 2	No solution. The only candidate is excluded.
28	2, -2	$x(x + 2) - 4(x - 2) = 16$, so $x^2 - 2x - 8 = 0$ and $(x - 4)(x + 2) = 0$ Candidates: 4 and -2	$x = 4$. The candidate -2 is excluded.

Notes for whoever is grading this

Problems 6, 11, 13 and 27 have no solution. The answer "no solution" is a complete answer and should score the same as a numerical one. A student who writes the excluded value as the answer has not finished the problem.

Problems 16, 17, 19 and 28 each lose one candidate. Look for the crossing out and the reason, not only the surviving value. In each of these the discarded root is a factor of the LCD, which is the pattern worth pointing out.

Problem 19 is the hole. The factor x - 2 cancels, so the left side equals x + 2 at every value except 2, and the graph is a line with one point missing rather than a curve with an asymptote. The value is still excluded. Watch for students who cancel first and then read the excluded values off the simplified expression, because that method loses 2 entirely.

Problem 21 is the one to talk about. The candidate -3 is not excluded, so it is not extraneous. It fails because a hose cannot take a negative number of hours. Students who label it extraneous have the right instinct and the wrong vocabulary.

Problem 24 is an identity. Both sides are the same expression, so every value works except the two excluded ones. A student who writes "no solution" has read 0 = 0 backwards.

Full guide, worked examples, and an interactive extraneous root visualizer: burketutoringinfremont.com/rational-equations-worksheet/