Rational Equation Word Problems: Setup Card

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Three formulas

- **Work:** rate = 1 ÷ time alone; part done = rate × time worked.
- **Motion:** time = distance ÷ speed. With a current c: $r + c$ one way, $r - c$ the other.
- **Shares and ratios:** share = total ÷ count; proportion = part ÷ whole, so $\frac{3}{4} = 75\%$.

The routine

1. Name the unknown with its units.
2. Build a table: one row per worker, leg, or group.
3. Fill the fraction column by dividing.
4. Turn the sentence that compares the rows into the equation.
5. List the excluded values, multiply by the LCD, solve.
6. Check in the equation, then in the table. Answer with units.

Phrases to equations

Words in the problem	What they tell you	Equation piece
"Working together, they finish in 2 hours"	The rates add to the rate together	$\frac{1}{a} + \frac{1}{b} = \frac{1}{2}$
"The drain empties it in d hours"	Subtract the drain's rate	$\frac{1}{a} - \frac{1}{d}$
"Hose B takes 4 hours longer than hose A"	Compare the times	B's time = $t + 4$
"Alone for 2 hours, then he joins"	T = total time: she works T h, he works $T - 2$ h	$\frac{T}{a} + \frac{T-2}{b} = 1$
"In the same time"	The times are equal	$\frac{9}{r+2} = \frac{5}{r-2}$
"The round trip takes 2 hours 30 minutes"	The times add, in hours	$\frac{15}{r} + \frac{15}{r-5} = \frac{5}{2}$
"Downstream" / "upstream"	The current adds / subtracts	$r + c$ and $r - c$
"Arrives 30 minutes sooner"	The times differ by $\frac{1}{2}$ hour	$\frac{90}{r} - \frac{90}{r+15} = \frac{1}{2}$
"Each share drops by $10"	Old share minus new share	$\frac{240}{n} - \frac{240}{n+2} = 10$
"Wins the next x games in a row"	Adds to the part and the whole	$\frac{9+x}{15+x}$
"Average speed for the round trip"	Total distance ÷ total time	$\frac{2d}{\frac{d}{10} + \frac{d}{r}}$

Two checks

- **Equation:** reject any candidate that makes an original denominator zero; it lies outside the equation's domain. Substitute the rest into the original equation.
- **Story:** times, speeds, and counts must be positive, counts must be whole, and a current must be slower than the boat. While a pipe and a drain both run, the filling rate must exceed the draining rate for the tank to keep filling. A candidate that fails here is nonviable.

Work estimate

Two workers who take a and b hours alone finish together in $T = \frac{ab}{a+b}$. That is less than the faster time alone and at least half of it.

Worked in one line

9 miles downstream and 5 upstream in the same time, current 2 mph: $\frac{9}{r+2} = \frac{5}{r-2} \rightarrow 9r - 18 = 5r + 10 \rightarrow r = 7$ mph.

Rational Equation Word Problems Worksheet

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Name _____ Date _____ Period _____

For each problem:

1. Name the unknown with its units.
2. Fill in the table. Parts A to C have the column headings printed; in Part D, write your own.
3. Write the equation, list the excluded values, and solve.
4. Check each candidate in the equation and in the table, then answer in a sentence with units.

Assume rates and speeds are constant, shared costs are divided equally, and quiz scores are equally weighted.

Part A · Work and rate problems

Problems 1 to 6. A worker's rate is 1 job divided by the time alone. Rates add when workers share a job, and a drain's rate is subtracted.

1. Maya can weed the school garden in 3 hours. Jordan can weed it in 6 hours. How long does the job take if they work together?

	Time alone	Rate	Time worked

Answer: _____

2. One printer finishes a batch of flyers in 45 minutes. With a second printer running at the same time, the batch takes 20 minutes. How long would the second printer take alone?

	Time alone	Rate	Time worked

Answer: _____

3. Hose B takes 4 hours longer than hose A to fill a pool. With both hoses running, the pool fills in 1 hour 30 minutes. How long does each hose take alone?

	Time alone	Rate	Time worked

Answer: _____

4. An inlet pipe fills a water tank in 6 hours. With the drain open as well, the tank takes 10 hours to fill. How long would the open drain take to empty a full tank?

	Time alone	Rate	Time worked

Answer: _____

5. Ana can paint a fence alone in 8 hours. She paints alone for 2 hours, then Ben joins her, and together they finish 2 hours later. How long would Ben take to paint the fence alone?

	Time alone	Rate	Time worked

Answer: _____

6. Three pumps running together empty a flooded basement in 2 hours. Pump A alone takes 6 hours and pump B alone takes 5 hours. How long would pump C take alone?

	Time alone	Rate	Time worked

Answer: _____

Part B · Distance, speed, and current

Problems 7 to 12. Time is distance divided by speed. A current or wind adds to the speed in one direction and subtracts in the other.

7. A cyclist rides 30 miles in the same time a runner covers 12 miles. The cyclist's average speed is 9 mph faster than the runner's. Find both speeds.

	Distance	Speed	Time

Answer: _____

8. A boat that goes 12 mph in still water travels 45 miles downstream in the same time it travels 27 miles upstream. How fast is the current?

	Distance	Speed	Time

Answer: _____

9. A small plane flies 600 miles with a tailwind in the same time it flies 480 miles into a headwind of the same speed. The wind is 20 mph. What is the plane's speed in still air?

	Distance	Speed	Time

Answer: _____

10. A kayaker paddles 6 miles upstream and 6 miles back down. The current is 2 mph, and the round trip takes 4 hours of paddling. What is her speed in still water?

	Distance	Speed	Time

Answer: _____

11. A car and a bus each travel 90 miles. The car averages 15 mph faster than the bus and arrives 30 minutes sooner. Find both speeds.

	Distance	Speed	Time

Answer: _____

12. A delivery driver travels 40 miles to a stop and 40 miles back along the same route. Traffic on the way back cuts the average speed by 10 mph. The driving time for the round trip is 1 hour 48 minutes. What was the speed on the way out?

	Distance	Speed	Time

Answer: _____

Part C · Ratios, percentages, and shares

Problems 13 to 18. A share is a total divided by a count. A proportion is a part divided by a whole (3/4 = 75%), and each new success adds to both.

13. A volleyball team has won 9 of its first 15 matches. How many matches in a row must it win to raise its winning percentage to 75%?

	Part or total	Whole or count	Fraction or share

Answer: _____

14. A softball player has 14 hits in 50 at-bats, a batting average of .280. Batting average is hits divided by at-bats, as defined in the MLB glossary. How many hits in consecutive at-bats would raise her average to .400?

	Part or total	Whole or count	Fraction or share

Answer: _____

15. A group splits the $600 cost of a party bus evenly. Five people drop out, and each remaining person pays $10 more. How many people were in the original group?

	Part or total	Whole or count	Fraction or share

Answer: _____

16. Last month a café spent $180 on bags of coffee beans. This month the price per bag is $3 higher, so the same $180 buys 3 fewer bags. How many bags did $180 buy last month?

	Part or total	Whole or count	Fraction or share

Answer: _____

17. A lab has 8 liters of a solution that is 25% acid. How many liters of pure acid must be added to make the solution 40% acid?

	Part or total	Whole or count	Fraction or share

Answer: _____

18. Kai's quizzes are equally weighted, and his average so far is 80. If he scores 100 on each of the next 3 quizzes, his average will rise to 85. How many quizzes has he taken so far?

	Part or total	Whole or count	Fraction or share

Answer: _____

Part D · Mixed problems

Problems 19 to 24. Numbers and reciprocals, two science formulas, an average speed problem, and one problem (23) where a positive candidate still fails the story.

19. The sum of a number and three times its reciprocal is 4. Find every such number.

Answer: _____

20. The denominator of a fraction is 5 more than its numerator. When 3 is added to both the numerator and the denominator, the result is $\frac{2}{3}$. Find the original fraction.

Answer: _____

21. Two resistors wired in parallel have a combined resistance R given by $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$. One resistor is 12 ohms and the combined resistance is 4 ohms. Find the other resistor.

Answer: _____

22. The thin lens equation $\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$ relates a lens's focal length to the object distance and the image distance (see HyperPhysics for the physics). A converging lens with focal length 12 cm forms a real image whose distance from the lens is three times the object distance. How far is the object from the lens?

Answer: _____

23. Two copiers running together finish a print job in 4 minutes. The faster copier alone takes 6 minutes less than the slower one. How long does each copier take alone?

Answer: _____

24. A cyclist rides out at 10 mph and returns on the same route. Her average speed for the whole round trip is 12 mph. How fast did she ride back?

Answer: _____

Challenge set · Problems 25 to 27

Show every step. Problem 27 ends with a short proof.

25. Return to problem 13. Show that no winning streak raises the volleyball team's percentage to exactly 100%. Then find the percentage after 30 straight wins, and describe what happens to the percentage as the streak gets longer.

26. Pipes A and B together fill a tank in 4 hours, pipes B and C together fill it in 6 hours, and pipes A and C together fill it in 5 hours. How long do all three pipes take together? How long does pipe A take alone?

27. A boat that goes 8 mph in still water travels 15 miles upstream and 15 miles back in 4 hours. (a) Find the speed of the current. (b) For a current c and a still-water speed r, show that when $0 < c < r$, a round trip of d miles each way takes longer than it would in still water.

Answer key

Each answer gives an equation and the answer, along with the relevant restrictions and story checks.

Part A · Work and rate problems

1. Let t = hours together. $\frac{1}{3} + \frac{1}{6} = \frac{1}{t}$. Excluded: $t \neq 0$. LCD $6t$: $2t + t = 6$, so $t = 2$. Estimate check: 2 hours is less than 3 hours and at least half of it.
   Answer: 2 hours

2. Let p = minutes for the second printer. $\frac{1}{45} + \frac{1}{p} = \frac{1}{20}$. Excluded: $p \neq 0$. LCD $180p$: $4p + 180 = 9p$, so $5p = 180$ and $p = 36$. Story check: 36 minutes is longer than the 20 minutes together.
   Answer: 36 minutes

3. Let t = hours for hose A, so hose B takes $t + 4$. 1 hour 30 minutes is $\frac{3}{2}$ hours, so the rate together is $\frac{2}{3}$: $\frac{1}{t} + \frac{1}{t+4} = \frac{2}{3}$. Excluded: 0, −4. LCD $3t(t + 4)$: $3(t + 4) + 3t = 2t(t + 4)$, so $6t + 12 = 2t^2 + 8t$ and $2t^2 + 2t - 12 = 0$. Divide by 2: $t^2 + t - 6 = 0$, which factors as $(t + 3)(t - 2) = 0$. The candidate −3 satisfies the equation but is a negative time, so it is nonviable.
   Answer: Hose A 2 hours, hose B 6 hours

4. Let d = hours for the drain. $\frac{1}{6} - \frac{1}{d} = \frac{1}{10}$. Excluded: $d \neq 0$. LCD $30d$: $5d - 30 = 3d$, so $d = 15$. Story check: the drain has to be slower than the pipe for the tank to fill, and 15 > 6.
   Answer: 15 hours

5. Let b = Ben's hours alone. Ana works 4 hours in all and does $\frac{4}{8}$ of the fence. Ben works 2 hours and does $\frac{2}{b}$. The parts add to one fence: $\frac{4}{8} + \frac{2}{b} = 1$. Excluded: $b \neq 0$. Then $\frac{2}{b} = \frac{1}{2}$, so $b = 4$.
   Answer: 4 hours

6. Let c = hours for pump C. $\frac{1}{6} + \frac{1}{5} + \frac{1}{c} = \frac{1}{2}$. Excluded: $c \neq 0$. LCD $30c$: $5c + 6c + 30 = 15c$, so $4c = 30$ and $c = \frac{15}{2}$.
   Answer: 7.5 hours (7 hours 30 minutes)

Part B · Distance, speed, and current

7. Let r = the runner's speed. $\frac{12}{r} = \frac{30}{r+9}$. Excluded: 0, −9. Cross multiply: $12(r + 9) = 30r$, so $108 = 18r$ and $r = 6$. Each travels for 2 hours.
   Answer: Runner 6 mph, cyclist 15 mph

8. Let c = the current. $\frac{45}{12+c} = \frac{27}{12-c}$. Excluded: 12, −12. $45(12 - c) = 27(12 + c)$ gives $540 - 45c = 324 + 27c$, so $72c = 216$ and $c = 3$. Story check: $0 < 3 < 12$. Each leg takes 3 hours.
   Answer: 3 mph

9. $\frac{600}{r+20} = \frac{480}{r-20}$. Excluded: 20, −20. $600(r - 20) = 480(r + 20)$ gives $600r - 12000 = 480r + 9600$, so $120r = 21600$ and $r = 180$. Each leg takes 3 hours.
   Answer: 180 mph

10. $\frac{6}{r-2} + \frac{6}{r+2} = 4$. Excluded: 2, −2. LCD $(r - 2)(r + 2)$: $6(r + 2) + 6(r - 2) = 4(r^2 - 4)$, so $12r = 4r^2 - 16$ and $4r^2 - 12r - 16 = 0$. Divide by 4: $r^2 - 3r - 4 = 0$, which factors as $(r - 4)(r + 1) = 0$. A speed of −1 is nonviable. Check: 3 hours up plus 1 hour back is 4 hours.
    Answer: 4 mph

11. Let r = the bus speed. The bus takes $\frac{1}{2}$ hour longer: $\frac{90}{r} - \frac{90}{r+15} = \frac{1}{2}$. Excluded: 0, −15. LCD $2r(r+15)$: $180(r+15) - 180r = r(r+15)$, so $2700 = r^2 + 15r$, or $r^2 + 15r - 2700 = 0$, which factors as $(r-45)(r+60) = 0$. A speed of −60 is nonviable. Times: 2 hours and 1.5 hours.

Answer: Bus 45 mph, car 60 mph

12. 1 hour 48 minutes is $\frac{9}{5}$ hours. $\frac{40}{r} + \frac{40}{r-10} = \frac{9}{5}$. Excluded: 0, 10. LCD $5r(r-10)$: $200(r-10) + 200r = 9r(r-10)$, so $400r - 2000 = 9r^2 - 90r$ and $9r^2 - 490r + 2000 = 0$, which factors as $(r-50)(9r-40) = 0$. The candidate $\frac{40}{9}$ satisfies the equation, but the return speed would be $\frac{40}{9} - 10$, which is negative, so it is nonviable. Check: 0.8 hour + 1 hour = 1.8 hours.

Answer: 50 mph out, 40 mph back

Part C · Ratios, percentages, and shares

13. $\frac{9+x}{15+x} = \frac{3}{4}$. Excluded: −15. $4(9+x) = 3(15+x)$, so $36 + 4x = 45 + 3x$ and $x = 9$. Check: $\frac{18}{24} = 0.75$.

Answer: 9 matches

14. Each hit is also an at-bat. $\frac{14+h}{50+h} = \frac{2}{5}$. Excluded: −50. $5(14+h) = 2(50+h)$, so $70 + 5h = 100 + 2h$ and $h = 10$. Check: $\frac{24}{60} = .400$.

Answer: 10 hits

15. $\frac{600}{n-5} - \frac{600}{n} = 10$. Excluded: 0, 5. LCD $n(n-5)$: $600n - 600(n-5) = 10n(n-5)$, so $3000 = 10n^2 - 50n$. Divide by 10: $n^2 - 5n - 300 = 0$, which factors as $(n-20)(n+15) = 0$. A group of −15 is nonviable. Check: \$30 each, then \$40 each.

Answer: 20 people

16. Let n = bags last month. $\frac{180}{n-3} - \frac{180}{n} = 3$. Excluded: 0, 3. LCD $n(n-3)$: $180n - 180(n-3) = 3n(n-3)$, so $540 = 3n^2 - 9n$. Divide by 3: $n^2 - 3n - 180 = 0$, which factors as $(n-15)(n+12) = 0$. A count of −12 bags is nonviable. Check: \$12 a bag, then \$15 a bag.

Answer: 15 bags

17. The solution holds $0.25 \times 8 = 2$ liters of acid. $\frac{2+x}{8+x} = \frac{2}{5}$. Excluded: −8. $5(2+x) = 2(8+x)$, so $3x = 6$ and $x = 2$. Check: $\frac{4}{10} = 40\%$.

Answer: 2 liters

18. Let n = quizzes so far, so his points total $80n$. $\frac{80n+300}{n+3} = 85$. Excluded: −3. $80n + 300 = 85n + 255$, so $5n = 45$ and $n = 9$. Check: $\frac{1020}{12} = 85$.

Answer: 9 quizzes

Part D · Mixed problems

19. $x + \frac{3}{x} = 4$. Excluded: 0. Multiply by x: $x^2 - 4x + 3 = 0$, so $(x-1)(x-3) = 0$. Both candidates check.

Answer: $x = 1$ or $x = 3$

20. Let n = the numerator, so the original fraction is $\frac{n}{n+5}$ and $n \neq -5$. $\frac{n+3}{n+8} = \frac{2}{3}$. Excluded from this equation: −8. $3(n+3) = 2(n+8)$, so $n = 7$, which is neither −5 nor −8. Check: $\frac{10}{15} = \frac{2}{3}$.

Answer: $\frac{7}{12}$

21. $\frac{1}{4} = \frac{1}{12} + \frac{1}{R_2}$. Excluded: $R_2 \neq 0$. LCD $12R_2$: $3R_2 = R_2 + 12$, so $R_2 = 6$.
Answer: 6 ohms

22. Let d = the object distance, so the real image is $3d$ from the lens. $\frac{1}{12} = \frac{1}{d} + \frac{1}{3d}$. Excluded: $d \neq 0$. LCD $12d$: $d = 12 + 4$, so $d = 16$. Check: $\frac{1}{16} + \frac{1}{48} = \frac{4}{48} = \frac{1}{12}$.
Answer: 16 cm, with the image at 48 cm

23. Let t = the slower copier's minutes, so the faster one takes $t - 6$. $\frac{1}{t} + \frac{1}{t-6} = \frac{1}{4}$. Excluded: 0, 6. LCD $4t(t-6)$: $4(t-6) + 4t = t(t-6)$, so $8t - 24 = t^2 - 6t$ and $t^2 - 14t + 24 = 0$, which factors as $(t - 12)(t - 2) = 0$. The candidate 2 satisfies the equation, but it gives the faster copier $2 - 6 = -4$ minutes, so it is nonviable.
Answer: Slower 12 minutes, faster 6 minutes

24. Let d = the one-way distance and r = the return speed. Average speed is total distance over total time: $\frac{2d}{\frac{d}{10} + \frac{d}{r}} = 12$. The d cancels, leaving $\frac{2}{\frac{1}{10} + \frac{1}{r}} = 12$, so $\frac{1}{10} + \frac{1}{r} = \frac{1}{6}$. Excluded: 0, −10. LCD $30r$: $3r + 30 = 5r$, so $r = 15$. The answer is not 14, because the slower leg takes more of the time. Challenge problem 27 on the complex rational expressions worksheet derives the general formula.
Answer: 15 mph

Challenge set

25. $\frac{9+x}{15+x} = 1$ gives $9 + x = 15 + x$, which reduces to $9 = 15$. No value of x works, so there is no solution. After 30 wins the record is $\frac{39}{45} \approx 86.7\%$. As x grows the fraction gets closer to 1 without reaching it, which is the horizontal asymptote $y = 1$ of $y = \frac{9+x}{15+x}$. The page on graphing rational functions covers horizontal asymptotes.
Answer: No streak reaches 100%

26. Let a, b, and c be the rates in tanks per hour. $a + b = \frac{1}{4}$, $b + c = \frac{1}{6}$, and $a + c = \frac{1}{5}$. Adding the three equations gives $2(a + b + c) = \frac{37}{60}$, so $a + b + c = \frac{37}{120}$. Pipe A: $a = \frac{37}{120} - \frac{1}{6} = \frac{17}{120}$.
Answer: All three: $\frac{120}{37}$ hours, about 3 hours 15 minutes. Pipe A: $\frac{120}{17}$ hours, about 7 hours 4 minutes

27. (a) $\frac{15}{8-c} + \frac{15}{8+c} = 4$. Excluded: 8, −8. LCD $(8 - c)(8 + c)$: $15(8 + c) + 15(8 - c) = 4(64 - c^2)$, so $240 = 256 - 4c^2$ and $c^2 = 4$. The candidate −2 is nonviable. (b) The round trip takes $\frac{d}{r-c} + \frac{d}{r+c} = \frac{2dr}{r^2 - c^2}$. Since $0 < c < r$, the denominator satisfies $0 < r^2 - c^2 < r^2$, so this is greater than $\frac{2dr}{r^2} = \frac{2d}{r}$, the still water time.
Answer: (a) 2 mph