Projectile Motion Reference Sheet

Level ground, platforms, and walls · g = 9.8 m/s^2 · air resistance neglected · AP Physics 1

Sort the problem into one of three cases first

Case	How to recognize it	Where the time comes from	Range formula valid?
	**1. Level ground.** Lands at the height it was launched from.	Symmetry: $T = 2v_{0y}/g$	Valid
	**2. Platform.** Lands higher or lower than it started.	The quadratic in t (Case 2 below)	Not valid
	**3. Barrier.** Asks about a height at a stated distance.	$t = d/v_{0x}$, or skip t with the trajectory equation	Not needed

Icon key: blue is the launch point and path, green is where the object lands or passes the barrier, orange is the barrier. Case 2 shows both directions: launched off a platform, and launched up onto a ledge.

Start every problem the same way

$v_{0x} = v_0 \cos\theta$ $\quad v_{0y} = v_0 \sin\theta$

$\Delta x = v_{0x} t$ $\quad (a_x = 0)$

$\Delta y = v_{0y} t - \frac{1}{2} g t^2$ $\quad v_y = v_{0y} - gt$

Put the origin at the launch point and take up as positive. Then $a_y = -g$, and Δy is the landing height minus the launch height.

Case 1: level ground

$T = 2v_0 \sin\theta / g$

$R = v_0^2 \sin(2\theta) / g$ $\quad$ greatest at $\theta = 45°$

$H = (v_0 \sin\theta)^2 / 2g$ $\quad H/R = \tan\theta / 4$

- θ and $90° - \theta$ give the same range. The steeper launch stays up longer and goes higher.
- The apex is halfway along the range, and the speed there, $v_0 \cos\theta$, is the smallest on the path.
- The object lands at its launch speed, θ below the horizontal.

Before you hand it in

- Origin and positive direction marked on the sketch.
- Sign of Δy checked against the sketch.
- Level ground shortcuts used only when the heights match.
- Root choice stated in words.
- Unrounded values carried until the final line.

Case 2: platforms

$4.9t^2 - v_{0y} t + \Delta y = 0$

$t = [\, v_{0y} \pm \sqrt{(v_{0y}^2 - 19.6\Delta y)} \,] / 9.8$

- **Landing lower ($\Delta y < 0$):** one positive root. Keep it.
- **Landing higher ($\Delta y > 0$):** if it gets that high, two positive roots. The smaller is the object passing that height on the way up; the larger is the landing. Check that the object is past the edge at that time.
- **Negative discriminant:** the object never reaches that height.
- Impact speed = $\sqrt{(v_0^2 - 2g\Delta y)}$, whatever the launch angle.
- It passes back through its launch height at speed v_0, θ below the horizontal.
- From height y_0 onto lower ground, the greatest distance comes from $\sin\theta = 1 / \sqrt{(2 + 2gy_0/v_0^2)}$, which is below 45°.

Case 3: walls and barriers

$y = y_0 + x \tan\theta - gx^2 / (2v_0^2 \cos^2\theta)$

clearance = y at the barrier − barrier height

$v_{min} = \sqrt{[\, gd^2 / (2\cos^2\theta\,(d \tan\theta - (h - y_0)))\,]}$

- Check that the barrier is closer than the landing point. If not, the object never reaches it.
- Report the clearance with its sign and units, not just "clears" or "hits".
- Angle window: with $T = \tan\theta$, the equation becomes a quadratic in T. Its two roots are the grazing angles, and every angle between them clears. No real roots means no angle works at that speed.
- Landing inside a boundary is a second, separate test.