Projectile Motion Answer Key

Full worked solutions for all 30 problems · AP Physics 1

$g = 9.8$ m/s^2, air resistance neglected, origin at the launch point with up positive. Values between steps are shown to five significant figures and carried unrounded into the next step. Final answers are given to two significant figures unless the problem calls for more.

Part A · Level ground (problems 1 to 10)



Launch and landing heights are equal. The time comes from symmetry, $T = 2v_{0y}/g$, and $R = v_0^2 \sin(2\theta)/g$ applies.

Blue: launch point and path. Green: landing point, at the launch height.

1 *A ball is kicked from level ground at 18 m/s at 35° above the horizontal. Find the time of flight, the range, and the apex height.*

1. Components: $v_{0x} = 18 \cos 35° = 14.745$ m/s and $v_{0y} = 18 \sin 35° = 10.324$ m/s.
2. Level ground, so the time of flight comes from symmetry: $T = 2v_{0y}/g = 2(10.324)/9.8 = 2.1070$ s.
3. Range: $R = v_{0x}T = (14.745)(2.1070) = 31.067$ m.
4. Apex height uses the vertical component only: $H = v_{0y}^2/2g = (10.324)^2/19.6 = 5.4384$ m.

Answer: T = 2.1 s, R = 31 m, H = 5.4 m

2 *Using the same launch speed of 18 m/s, find the two launch angles that produce a range of exactly 30 m.*

1. Start from $R = v_0^2 \sin(2\theta)/g$ and solve for the sine: $\sin(2\theta) = Rg/v_0^2 = (30)(9.8)/18^2 = 294/324 = 0.90741$.
2. Between 0° and 180° two angles have this sine: $2\theta = 65.15°$ or $2\theta = 180° - 65.15° = 114.85°$.
3. Halve each: $\theta = 32.57°$ or $\theta = 57.43°$.
4. Check: $32.57° + 57.43° = 90°$, so the two answers are a complementary pair, as level ground ranges must be.

Answer: θ = 33° or θ = 57°

3 *A projectile leaves level ground at 22 m/s at 55°. Find its speed and its direction 1.2 s after launch, and state whether it is rising or falling at that moment.*

1. Horizontal velocity never changes: $v_x = 22 \cos 55° = 12.619$ m/s.
2. Vertical velocity at 1.2 s: $v_y = 22 \sin 55° - 9.8(1.2) = 18.021 - 11.76 = +6.2613$ m/s.
3. Speed: $\sqrt{(12.619^2 + 6.2613^2)} = 14.087$ m/s.
4. Direction: $\arctan(6.2613/12.619) = 26.4°$ above the horizontal.
5. v_y is positive, so the projectile is still rising. The apex comes at $t = v_{0y}/g = 1.84$ s.

Answer: 14 m/s at 26° above the horizontal, still rising

4 *A projectile launched at 30° above level ground travels 40 m before landing. Find its launch speed.*

1. Rearrange $R = v_0^2 \sin(2\theta)/g$ for the speed: $v_0 = \sqrt{(Rg / \sin 2\theta)}$.
2. Substitute: $v_0 = \sqrt{((40)(9.8)/\sin 60°)} = \sqrt{(392/0.86603)} = \sqrt{452.64} = 21.275$ m/s.

Answer: v_0 = 21 m/s

5 *Two balls leave level ground at 20 m/s, one at 25° and one at 65°. Compare their ranges, their times of flight, and their apex heights, with numbers.*

1. Components at 25°: $v_{0x} = 18.126$ m/s, $v_{0y} = 8.4524$ m/s. At 65° the two values swap.
2. Ranges: $R = 2v_{0x}v_{0y}/g = 2(18.126)(8.4524)/9.8 = 31.267$ m for both angles.
3. Times: $T = 2v_{0y}/g$ gives 1.7250 s at 25° and 3.6992 s at 65°.
4. Apex heights: $H = v_{0y}^2/19.6$ gives 3.6450 m at 25° and 16.763 m at 65°.
5. The steep launch reaches 4.6 times the height for the same range.

Answer: R = 31 m for both; T = 1.7 s and 3.7 s; H = 3.6 m and 17 m

6 *Show that on level ground the apex height equals one quarter of the range only when the launch angle is 45°. Start from H/R and simplify.*

1. Write both quantities with components: $H = v_{0y}^2/2g$ and $R = 2v_{0x}v_{0y}/g$.
2. Divide: $H/R = [v_{0y}^2/2g] \div [2v_{0x}v_{0y}/g] = v_{0y}/(4v_{0x})$.
3. Since $v_{0y}/v_{0x} = \tan\theta$, $H/R = \tan\theta/4$.
4. Set $H/R = 1/4$: $\tan\theta = 1$, so $\theta = 45°$. No other angle between 0° and 90° gives $\tan\theta = 1$.
5. Check with a different angle: at 30°, $H/R = \tan 30°/4 = 0.144$, not 0.25.

Answer: H/R = tanθ/4, which equals 1/4 only at θ = 45°

7 *A projectile on level ground has a time of flight of 2.4 s and a range of 36 m. Find the vertical and horizontal components of its launch velocity, then its launch speed, launch angle, and apex height.*

1. From $T = 2v_{0y}/g$: $v_{0y} = gT/2 = 9.8(2.4)/2 = 11.76$ m/s.
2. Horizontal speed is constant: $v_{0x} = R/T = 36/2.4 = 15$ m/s.
3. Launch speed: $v_0 = \sqrt{(15^2 + 11.76^2)} = 19.060$ m/s.
4. Launch angle: $\theta = \arctan(11.76/15) = 38.10°$.
5. Apex height: $H = v_{0y}^2/2g = (11.76)^2/19.6 = 7.0560$ m.

Answer: v_{0y} = 11.76 m/s, v_{0x} = 15 m/s, v_0 = 19 m/s at 38°, H = 7.1 m

8 *Two projectiles leave level ground at 16 m/s, at 20° and at 70°. Which lands first, and by how many seconds?*

1. $T = 2v_0 \sin\theta/g$. At 20°: $T = 2(16 \sin 20°)/9.8 = 1.1168$ s.
2. At 70°: $T = 2(16 \sin 70°)/9.8 = 3.0684$ s.
3. Difference: $3.0684 - 1.1168 = 1.9516$ s.
4. The angles are complementary, so both land at the same place: $R = 16^2 \sin 40°/9.8 = 16.791$ m.

Answer: The 20° launch lands first, by 2.0 s

9 *A punt has a hang time of 4.2 s and travels 52 m downfield before being caught at the height it was kicked. Find the launch speed, the launch angle, and the apex height.*

1. Caught at the kick height, so this is level ground: $v_{0y} = gT/2 = 9.8(4.2)/2 = 20.58$ m/s.
2. $v_{0x} = R/T = 52/4.2 = 12.381$ m/s.
3. $v_0 = \sqrt{(12.381^2 + 20.58^2)} = 24.017$ m/s at $\theta = \arctan(20.58/12.381) = 58.97°$.
4. $H = (20.58)^2/19.6 = 21.609$ m.

Answer: v_0 = 24 m/s at 59°, H = 22 m

10 *At what point on a level ground trajectory is the speed smallest, and why? Find that speed for a launch of 24 m/s at 50°.*

1. Speed is $\sqrt{(v_x^2 + v_y^2)}$. The horizontal part v_x never changes.
2. The vertical part shrinks to zero at the apex and grows again on the way down, so the speed is smallest at the apex.
3. At the apex the speed equals $v_{0x} = 24 \cos 50° = 15.427$ m/s.
4. The speed is never zero unless the launch was straight up.

Answer: At the apex, 15 m/s

Part B · Platforms (problems 1 to 10)



Launch and landing heights differ. Solve $4.9t^2 - v_{0y}t + \Delta y = 0$ for the time and say which root you keep. The range formula does not apply.

Left: launched off a platform to lower ground. Right: launched up onto a ledge. Blue: launch and path. Green: landing.

1 *A ball leaves the edge of a 12 m roof at 14 m/s directed 25° above the horizontal. Find the time to reach the ground, the horizontal distance, and the impact speed.*

1. Origin at the launch point, up positive: $\Delta y = -12$ m. Components: $v_{0x} = 12.688$ m/s, $v_{0y} = 5.9167$ m/s.
2. Vertical equation: $-12 = 5.9167t - 4.9t^2$, so $4.9t^2 - 5.9167t - 12 = 0$.
3. Quadratic formula: $t = [5.9167 \pm \sqrt{(5.9167^2 + 4(4.9)(12))}]/9.8 = [5.9167 \pm 16.438]/9.8$. The roots are 2.2811 s and −1.0736 s; keep the positive one.
4. Horizontal distance: $\Delta x = (12.688)(2.2811) = 28.943$ m.
5. Impact speed: $\sqrt{(v_0^2 + 2gh)} = \sqrt{(14^2 + 2(9.8)(12))} = 20.765$ m/s, directed $\arctan(16.438/12.688) = 52.3°$ below the horizontal.

Answer: t = 2.3 s, Δx = 29 m, impact speed 21 m/s

2 *The same ball is thrown from the same roof at 14 m/s but directed 25° below the horizontal. Find the time and the horizontal distance, and compare the impact speed with the previous answer.*

1. Now $v_{0y} = -5.9167$ m/s. The equation becomes $4.9t^2 + 5.9167t - 12 = 0$.
2. $t = [-5.9167 + \sqrt{((-5.9167)^2 + 4(4.9)(12))}]/9.8 = [-5.9167 + 16.438]/9.8 = 1.0736$ s.
3. $\Delta x = (12.688)(1.0736) = 13.622$ m.
4. Impact speed is again $\sqrt{(14^2 + 2(9.8)(12))} = 20.765$ m/s. That expression does not contain the angle.
5. Compared with problem 1, this throw lands 1.2 s sooner and 15 m closer at the same speed.

Answer: t = 1.1 s, Δx = 14 m, impact speed 21 m/s (same as problem 1)

3 *A marble leaves a 9.0 m ledge horizontally at 7.5 m/s. Find the time in the air, the horizontal distance, the impact speed, and the angle below the horizontal at which it lands.*

1. Horizontal launch, so $v_{0y} = 0$ and the fall time depends on height only: $t = \sqrt{(2h/g)} = \sqrt{(2(9.0)/9.8)} = 1.3553$ s.
2. $\Delta x = 7.5(1.3553) = 10.164$ m.
3. Vertical velocity at impact: $v_y = -9.8(1.3553) = -13.282$ m/s.
4. Impact speed: $\sqrt{(7.5^2 + 13.282^2)} = 15.253$ m/s.
5. Impact angle: $\arctan(13.282/7.5) = 60.5°$ below the horizontal.

Answer: t = 1.4 s, Δx = 10 m, 15 m/s at 61° below the horizontal

4 *A ball is launched from the ground at 20 m/s at 60° and lands on top of a platform 8.0 m high. Find both times at which it is 8.0 m above the ground, state which is the landing, and find the horizontal distance and the landing speed.*

1. Components: $v_{0x} = 10$ m/s, $v_{0y} = 17.321$ m/s. Landing above the launch point, so $\Delta y = +8.0$ m.
2. $8.0 = 17.321t - 4.9t^2$, so $4.9t^2 - 17.321t + 8.0 = 0$.
3. Discriminant: $17.321^2 - 4(4.9)(8.0) = 143.20$, $\sqrt{} = 11.967$. Roots: $t = (17.321 \pm 11.967)/9.8 = 0.54632$ s and 2.9885 s.
4. Both are positive. The earlier root is the ball passing 8.0 m on the way up, before the platform is under it. The later root, 2.9885 s, is the landing.
5. $\Delta x = (10)(2.9885) = 29.885$ m.
6. At landing $v_y = 17.321 - 9.8(2.9885) = -11.967$ m/s, so the speed is $\sqrt{(10^2 + 11.967^2)} = 15.595$ m/s.

Answer: t = 0.55 s and 3.0 s; lands at the later time, Δx = 30 m, 16 m/s

5 *A ball is thrown from the ground at 15 m/s at 55° toward a garage roof 3.0 m high. What is the greatest distance the near edge of the roof can be if the ball is to land on the roof rather than hit the wall?*

1. The ball is at roof height twice. The far limit for the edge is where it comes back down to 3.0 m, which is the later root of $\Delta y = +3.0$ m.
2. Components: $v_{0x} = 8.6036$ m/s, $v_{0y} = 12.287$ m/s.
3. $4.9t^2 - 12.287t + 3.0 = 0$ gives $t = 0.27412$ s and 2.2335 s. Keep 2.2335 s.
4. $\Delta x = (8.6036)(2.2335) = 19.216$ m.
5. If the edge were any farther, the ball would already be below roof height when it got there, and it would hit the wall.

Answer: About 19 m

6 *A ball is thrown at 11 m/s at 40° above the horizontal from a balcony 5.0 m above the ground. Find the maximum height above the ground, the time to reach the ground, the horizontal distance, and the impact speed.*

1. Components: $v_{0x} = 8.4265$ m/s, $v_{0y} = 7.0707$ m/s.
2. Rise above the balcony: $v_{0y}^2/2g = (7.0707)^2/19.6 = 2.5507$ m. Maximum height above the ground = $5.0 + 2.5507 = 7.5507$ m.
3. To the ground $\Delta y = -5.0$ m: $4.9t^2 - 7.0707t - 5.0 = 0$ gives $t = 1.9629$ s (the other root is negative).
4. $\Delta x = (8.4265)(1.9629) = 16.540$ m.
5. Impact speed: $\sqrt{(11^2 + 2(9.8)(5.0))} = 14.799$ m/s.

Answer: H = 7.6 m, t = 2.0 s, Δx = 17 m, 15 m/s

7 *Two balls are thrown from a 10 m platform at 16 m/s, one at 30° above the horizontal and one at 60°. Find each impact speed, each time in the air, and each horizontal distance. Comment on the pattern.*

1. Impact speed for both: $\sqrt{(v_0^2 + 2gh)} = \sqrt{(16^2 + 2(9.8)(10))} = \sqrt{452} = 21.260$ m/s.
2. At 30°: $v_{0y} = 8$ m/s; $4.9t^2 - 8t - 10 = 0$ gives $t = 2.4617$ s and $\Delta x = (13.856)(2.4617) = 34.110$ m.
3. At 60°: $v_{0y} = 13.856$ m/s; $4.9t^2 - 13.856t - 10 = 0$ gives $t = 3.4239$ s and $\Delta x = (8)(3.4239) = 27.391$ m.
4. Pattern: the steeper throw stays up longer and lands closer, but both arrive at the same speed.

Answer: Both 21 m/s; 2.5 s and 3.4 s; 34 m and 27 m

8 *A ball rolls off a table 0.95 m high and lands 1.35 m from the base of the table. Find the time in the air, the speed it left the table, and the impact speed.*

1. It leaves horizontally, so the fall time comes from the height: $t = \sqrt{(2(0.95)/9.8)} = 0.44032$ s.
2. Horizontal speed: $v_0 = \Delta x/t = 1.35/0.44032 = 3.0660$ m/s.
3. Vertical speed at impact: $9.8(0.44032) = 4.3151$ m/s.
4. Impact speed: $\sqrt{(3.0660^2 + 4.3151^2)} = 5.2934$ m/s.

Answer: t = 0.44 s, v_0 = 3.1 m/s, impact 5.3 m/s

9 *A ball is launched at 18 m/s at 32° from a platform 6.0 m above the ground. Find its velocity and horizontal position at the moment it returns to launch height, then find the time and horizontal distance when it reaches the ground.*

1. Components: $v_{0x} = 15.265$ m/s, $v_{0y} = 9.5385$ m/s.
2. Up to launch height the motion is symmetric: $t = 2v_{0y}/g = 2(9.5385)/9.8 = 1.9466$ s and $\Delta x = (15.265)(1.9466) = 29.715$ m.
3. At that moment it moves at the launch speed, 18 m/s, directed 32° below the horizontal.
4. To the ground, $\Delta y = -6.0$ m: $4.9t^2 - 9.5385t - 6.0 = 0$ gives $t = 2.4470$ s.
5. $\Delta x = (15.265)(2.4470) = 37.354$ m, arriving at $\sqrt{(18^2 + 2(9.8)(6.0))} = 21.014$ m/s.

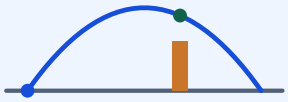
Answer: At launch height: 1.9 s, 30 m, 18 m/s at 32° below. Ground: 2.4 s, 37 m

10 *A ramp launches an object at 21 m/s at 28° from a lip 4.0 m above the ground onto a landing pad 1.5 m high. Find the time in the air, the horizontal distance traveled, the maximum height above the ground, and the impact speed.*

1. Δy = landing height − launch height = $1.5 - 4.0 = -2.5$ m.
2. Components: $v_{0x} = 18.542$ m/s, $v_{0y} = 9.8589$ m/s.
3. $4.9t^2 - 9.8589t - 2.5 = 0$ gives $t = 2.2398$ s (the other root is negative).
4. $\Delta x = (18.542)(2.2398) = 41.530$ m.
5. Maximum height above the ground: $4.0 + (9.8589)^2/19.6 = 8.9591$ m.
6. Impact speed: $\sqrt{(21^2 + 2(9.8)(2.5))} = \sqrt{490} = 22.136$ m/s.

Answer: t = 2.2 s, Δx = 42 m, H = 9.0 m, 22 m/s

Part C · Walls and barriers (problems 1 to 10)



Find the height at the barrier, either from $t = d/v_{0x}$ or from the trajectory equation, then report the clearance with its sign.

Orange: the barrier. Green: the object as it passes the barrier. Blue: launch and path.

1 *A free kick leaves the ground at 18 m/s at 25°. A defensive wall 2.0 m tall stands 9.15 m away. Find the height of the ball at the wall and state whether it clears.*

1. Components: $v_{0x} = 16.314$ m/s, $v_{0y} = 7.6071$ m/s.
2. Time to reach the wall: $t = d/v_{0x} = 9.15/16.314 = 0.56088$ s.
3. Height then: $y = v_{0y}t - 4.9t^2 = 4.2667 - 1.5415 = 2.7252$ m.
4. Clearance: $2.7252 - 2.0 = +0.73$ m.

Answer: 2.73 m at the wall; clears by 0.73 m

2 *For the same kick geometry, find the minimum launch speed at 25° that would just clear the 2.0 m wall at 9.15 m.*

1. Set the height at the wall equal to the wall height and solve the trajectory equation for v_0: $v_{min} = \sqrt{[gd^2 / (2\cos^2\theta\,(d\tan\theta - h))]}$.
2. Pieces: $d\tan\theta = 9.15\tan 25° = 4.2667$ m, $\cos^2 25° = 0.82139$, $gd^2 = 9.8(9.15)^2 = 820.48$.
3. $v_{min} = \sqrt{[820.48 / (2(0.82139)(4.2667 - 2.0))]} = \sqrt{[820.48/3.7237]} = 14.844$ m/s.
4. The actual kick at 18 m/s is 3.2 m/s above this.

Answer: v_{min} = 15 m/s

3 *A ball is struck from level ground at 24 m/s at 35°. A fence 7.0 m tall stands 40 m away. Does the ball clear it, and by how much?*

1. Use the trajectory equation with $x = 40$ m: $y = x\tan\theta - gx^2/(2v_0^2\cos^2\theta)$.
2. First term: $40\tan 35° = 28.008$ m.
3. Second term: $9.8(1600)/(2(576)(0.67101)) = 20.285$ m.
4. $y = 28.008 - 20.285 = 7.7238$ m, so the clearance is $7.7238 - 7.0 = +0.72$ m.

Answer: Yes, by 0.72 m

4 *The same ball as problem 3 continues past the fence. A second barrier 4.0 m tall stands 50 m from the launch point. Does the ball clear that one?*

1. First check it is still airborne: $R = 24^2 \sin 70°/9.8 = 55.231$ m, which is beyond 50 m.
2. $y = 50\tan 35° - 9.8(2500)/(2(576)(0.67101)) = 35.010 - 31.695 = 3.3158$ m.
3. Clearance: $3.3158 - 4.0 = -0.68$ m. Negative, so the ball strikes the barrier.

Answer: No. It is 0.68 m too low and hits the barrier

5 *A volleyball is struck at 10 m/s at 30° from a height of 2.2 m. The net is 2.43 m high and stands 4.5 m away, and the court boundary is 9.0 m from the point of contact. Determine whether the ball clears the net, then whether it lands in.*

1. Two separate tests: the height at the net, then the landing distance.
2. At the net: $y = 2.2 + 4.5 \tan 30° - 9.8(20.25)/(2(100)(0.75)) = 2.2 + 2.5981 - 1.3230 = 3.4751$ m. Clearance over 2.43 m: +1.05 m.
3. Landing on the floor: $\Delta y = -2.2$ m and $v_{0y} = 5$ m/s, so $4.9t^2 - 5t - 2.2 = 0$ gives $t = 1.3524$ s.
4. $\Delta x = (8.6603)(1.3524) = 11.712$ m, which is 2.7 m beyond the 9.0 m boundary.

Answer: Clears the net by 1.05 m, but lands out at 11.7 m

6 *A golf ball leaves the tee at 34 m/s at 28°. A tree 14 m tall stands 55 m down the fairway. Does the ball clear the tree?*

1. First term: $55 \tan 28° = 29.244$ m.
2. Second term: $9.8(3025)/(2(1156)(0.77960)) = 16.447$ m.
3. $y = 29.244 - 16.447 = 12.797$ m. Clearance: $12.797 - 14 = -1.20$ m.

Answer: No. It is 1.2 m too low

7 *For the shot in problem 6, find the minimum launch speed at 28° that would clear the tree.*

1. $v_{min} = \sqrt{[gd^2 / (2\cos^2\theta\,(d \tan\theta - h))]}$ with $d = 55$ m and $h = 14$ m.
2. $= \sqrt{[9.8(3025) / (2(0.77960)(29.244 - 14))]} = \sqrt{[29645/23.768]} = 35.316$ m/s.
3. That is 1.3 m/s more than the 34 m/s shot.

Answer: v_{min} = 35 m/s

8 *A ball is launched at 20 m/s from level ground toward a wall 6.0 m tall standing 25 m away. Find the two launch angles at which the ball just grazes the top of the wall, and state the range of angles that clear it.*

1. Use $1/\cos^2\theta = 1 + \tan^2\theta$ in the trajectory equation and write $T = \tan\theta$: $y = dT - (gd^2/2v_0^2)(1 + T^2)$.
2. Here $gd^2/2v_0^2 = 9.8(625)/800 = 7.6563$. Setting $y = 6.0$: $7.6563T^2 - 25T + 13.656 = 0$.
3. Discriminant: $25^2 - 4(7.6563)(13.656) = 206.78$, $\sqrt{} = 14.380$.
4. $T = (25 \pm 14.380)/(2 \times 7.6563) = 0.69357$ or 2.5717.
5. $\theta = \arctan T = 34.7°$ or $68.8°$. Any launch angle between these clears the wall.

Answer: Grazing at 34.7° and 68.8°; clears for angles between them

9 *A ball is launched at 26 m/s toward a wall 7.0 m tall standing 32 m away, first at 25° and then at 35°. Find the clearance in each case and comment on the size of the change.*

1. At 25°: y = 32 tan 25° − 9.8(1024)/(2(676)(0.82139)) = 14.922 − 9.0365 = 5.8854 m. Clearance: −1.11 m, so it hits.
2. At 35°: y = 32 tan 35° − 9.8(1024)/(2(676)(0.67101)) = 22.407 − 11.062 = 11.345 m. Clearance: +4.34 m.
3. Ten degrees of launch angle changes the height at the wall by 5.5 m, so a small error in the angle or the components can change the verdict.

Answer: 25°: −1.1 m (hits). 35°: +4.3 m (clears)

10 *A ball is launched from level ground at 20 m/s at 35°. It must clear a wall 4.0 m tall standing 20 m away and then land before a boundary line 45 m from the launch point. Check both conditions.*

1. Condition 1, the wall: y = 20 tan 35° − 9.8(400)/(2(400)(0.67101)) = 14.004 − 7.3024 = 6.7017 m. Clearance: +2.70 m.
2. Condition 2, the landing: $R = v_0^2 \sin(2\theta)/g = 20^2 \sin 70°/9.8 = 38.355$ m, which is 6.6 m inside the 45 m line.
3. Both conditions are met.

Answer: Clears by 2.7 m and lands at 38.4 m, inside the line