Domain Restrictions & Excluded Values Worksheet

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Name _____ Date _____ Period _____

Conditions to check

Denominator: the complete denominator $\neq 0$	Even root: radicand ≥ 0	Root that is the whole denominator or a factor of it: radicand > 0
Logarithm: argument > 0	Negative exponent: base $\neq 0$	Odd roots, polynomials, $\lvert x \rvert$: none of their own

Write every domain in interval notation. Shade it on the number line and label the endpoints you use.

Part A · Excluded values of rational expressions

List every excluded value. In problem 4, also simplify the expression.

1. $\dfrac{8}{x + 6}$

Excluded values: _____

2. $\dfrac{x - 1}{x^2 - 25}$

Excluded values: _____

3. $\dfrac{x^2 - 1}{x^2 - 10x + 25}$

Excluded values: _____

4. $\dfrac{x + 4}{x^2 + 2x - 8}$

Excluded values: _____

5. $\dfrac{3x}{2x^2 - 7x + 3}$

Excluded values: _____

6. $\dfrac{x^2 - 4}{x^3 - 9x}$

Excluded values: _____

Part B · Even and odd roots

Write the domain of each function in interval notation.

7. $f(x) = \sqrt{x - 4}$



Domain: _____

8. $f(x) = \sqrt[4]{10 - 2x}$



Domain: _____

9. $f(x) = \sqrt[3]{x - 8}$



Domain: _____

10. $f(x) = \sqrt{x^2 - 16}$



Domain: _____

11. $f(x) = \dfrac{5}{\sqrt{3 - x}}$



Domain: _____

12. $f(x) = \sqrt{x^3 - 4x}$



Domain: _____

Part C · Combined restrictions, exponents, and logarithms

Write the domain of each function in interval notation. Here log means base 10.

13. $f(x) = \dfrac{\sqrt{x + 3}}{x - 2}$



Domain: _____

14. $f(x) = (x + 2)(x^2 + 9)^{-1}$



Domain: _____

15. $f(x) = \dfrac{\sqrt{6 - x}}{x^2 - 9}$



Domain: _____

16. $f(x) = \ln x + \sqrt{5 - x}$



Domain: _____

17. $f(x) = (x - 1)^{-2} + \sqrt{x}$



Domain: _____

18. $f(x) = \log(x^2 - 4x)$

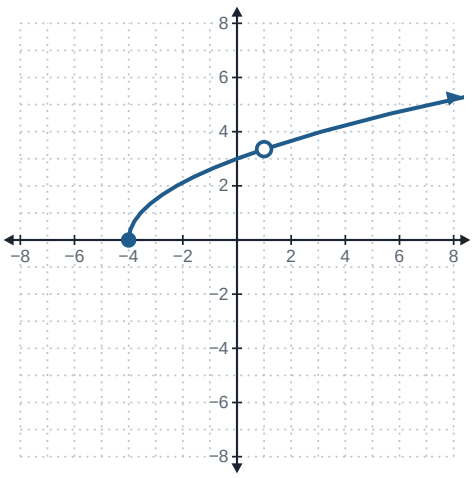


Domain: _____

Part D · Graphs, operations, composites, and context

Write each domain in interval notation.

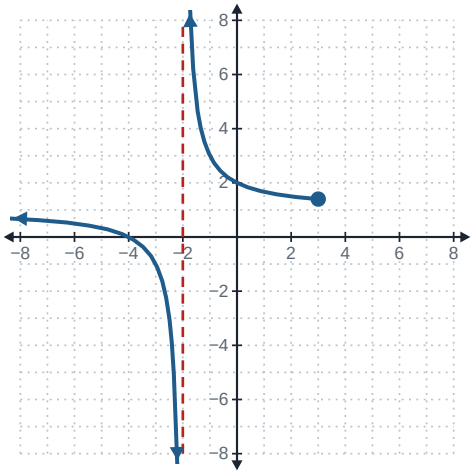
19. State the domain of the function graphed below.



———	Graph of the function
●	Closed dot: the point is on the graph
○	Open circle: the point is not on the graph
——→	Arrowhead: the graph continues

Domain: _____

20. State the domain of the function graphed below.



———	Graph of the function
●	Closed dot: the point is on the graph
- - -	Dashed line: vertical asymptote
——→	Arrowhead: the graph continues

Domain: _____

21. Let $f(x) = x - 5$ and $g(x) = \sqrt{x}$. Find the domain of $(f/g)(x)$.



Domain: _____

22. Let $f(x) = \dfrac{1}{x - 2}$ and $g(x) = \sqrt{x + 1}$. Find the domain of $f(g(x))$.



Domain: _____

23. Let $f(x) = x^2$ and $g(x) = \sqrt{x - 1}$. Simplify $f(g(x))$ and state its domain.



Domain: _____

24. A rectangle has a perimeter of 40 cm and a width of w cm. Its area is $A(w) = w(20 - w)$. State the domain of A in this context.



Domain: _____

Challenge problems

Show your conditions and your sign chart.

25. Find the domain of $f(x) = \sqrt{\dfrac{x+3}{x-1}}$. Explain why -3 is included and 1 is not.



Answer: _____

26. Write a function whose domain is $[-2, 5) \cup (5, \infty)$. Then explain which part of your function creates each piece of the domain.



Answer: _____

27. Let $f(x) = \dfrac{1}{x-2}$ and $g(x) = \sqrt{x+1}$. Find the domain of $g(f(x))$. Compare it with your answer to problem 22.



Answer: _____

Answer key

Each domain is shaded on a number line. The marked points are spaced evenly, not to scale. Closed dots are included; open circles are not.

Part A

1. $x \neq -6$

Domain: $(-\infty, -6) \cup (-6, \infty)$



2. $x \neq -5, 5$

Domain: $(-\infty, -5) \cup (-5, 5) \cup (5, \infty)$



3. $x \neq 5$ (the factor $(x - 5)^2$ gives one value)

Domain: $(-\infty, 5) \cup (5, \infty)$



4. $x \neq -4, 2$; simplifies to $\dfrac{1}{x - 2}$, $x \neq -4, 2$

Domain: $(-\infty, -4) \cup (-4, 2) \cup (2, \infty)$



5. $x \neq \dfrac{1}{2}$, 3, from $(2x - 1)(x - 3)$

Domain: $(-\infty, \dfrac{1}{2}) \cup (\dfrac{1}{2}, 3) \cup (3, \infty)$



6. $x \neq -3, 0, 3$, from $x(x - 3)(x + 3)$; ± 2 are allowed

Domain: $(-\infty, -3) \cup (-3, 0) \cup (0, 3) \cup (3, \infty)$



Part B

7. $[4, \infty)$



8. $(-\infty, 5]$



9. $(-\infty, \infty)$, since a cube root accepts negatives



10. $(-\infty, -4] \cup [4, \infty)$



11. $(-\infty, 3)$



12. $[-2, 0] \cup [2, \infty)$, from the sign chart of $x(x - 2)(x + 2)$



Part C

13. $[-3, 2) \cup (2, \infty)$



14. $(-\infty, \infty)$; the base $x^2 + 9$ has no real zeros



15. $(-\infty, -3) \cup (-3, 3) \cup (3, 6]$



16. $(0, 5]$



17. $[0, 1) \cup (1, \infty)$



18. $(-\infty, 0) \cup (4, \infty)$, from $x(x - 4) > 0$



Part D

19. $[-4, 1) \cup (1, \infty)$



20. $(-\infty, -2) \cup (-2, 3]$



21. $(0, \infty)$; g needs $x \geq 0$ and $g(x) \neq 0$ removes 0



22. $[-1, 3) \cup (3, \infty)$; $\sqrt{x + 1} \neq 2$ removes 3



23. $f(g(x)) = x - 1$, domain $[1, \infty)$



24. $(0, 20)$; both w and $20 - w$ are side lengths, so both are positive



Challenge problems

25. $(-\infty, -3] \cup (1, \infty)$. The inner denominator needs $x \neq 1$, and the radicand needs $\frac{x + 3}{x - 1} \geq 0$. A sign chart with cuts at -3 and 1 shows the fraction is positive for $x < -3$ and $x > 1$. At $x = -3$ the fraction equals 0, and $\sqrt{0} = 0$ is real, so -3 is included. At $x = 1$ the fraction is undefined, so 1 is excluded.



26. One answer is $f(x) = \frac{\sqrt{x + 2}}{x - 5}$. The square root requires $x \geq -2$, which starts the domain at -2 with a bracket, and the denominator removes 5. Grade other answers on whether their domain matches $[-2, 5) \cup (5, \infty)$; for example $\frac{1}{x - 5} + \sqrt[4]{x + 2}$ also works.



27. $(-\infty, 1] \cup (2, \infty)$. The inside function f needs $x \neq 2$. The outside function g needs $f(x) + 1 \geq 0$, which is $\frac{1}{x - 2} + 1 = \frac{x - 1}{x - 2} \geq 0$. A sign chart with cuts at 1 and 2 gives $x \leq 1$ or $x > 2$. Problem 22 had domain $[-1, 3) \cup (3, \infty)$, so reversing the order of a composition can change the domain completely.

