# Domain Restrictions & Excluded Values

Burke Tutoring in Fremont · one page reference

## FIND THE DOMAIN IN FOUR STEPS

1. Mark every denominator, even root, logarithm, and negative or fraction exponent.
2. Write one condition for each: $\neq 0$, $\geq 0$, or $> 0$.
3. Solve each condition. Use a sign chart when a radicand or log argument is a polynomial or a fraction.
4. Keep the numbers that pass every condition. Write the result in interval notation.

## CONDITIONS

- Denominator: the complete denominator $\neq 0$
- Even root: radicand $\geq 0$
- Root that is the whole denominator or a factor of it: the two rules combine to radicand $> 0$
- Logarithm: argument $> 0$
- Negative exponent: base $\neq 0$
- Polynomials, odd roots, $|x|$, $2^x$: no condition of their own

## INTERVAL NOTATION

- [ ] included, ( ) not included
- $-\infty$ and $\infty$ take parentheses
- $\cup$ joins pieces, listed left to right
- $x \neq a$ becomes $(-\infty, a) \cup (a, \infty)$

## READING A GRAPH

- Include every x-coordinate that has an included point
- Closed dot: an included point. At the end of a branch, use a bracket
- Open circle: that point is removed. Its x-value is out unless another included point shares it
- Vertical asymptote: the branch has no point at that x-value
- Use $\pm\infty$ only where a branch extends arbitrarily far left or right

## RATIONAL EXPRESSIONS AND COMPOSITES

- Take excluded values from the original denominator, before canceling. A factor that cancels completely leaves a hole; a factor that remains leaves a vertical asymptote.
- Numerator zeros are allowed. A denominator such as $x^2 + 9$ with no real zeros excludes nothing.
- For f(g(x)): x must be in the domain of g, and g(x) must be in the domain of f. Find the domain before simplifying.

## EXAMPLE: COMBINING RESTRICTIONS

$f(x) = \dfrac{\sqrt{x + 5}}{x^2 - 4}$ Domain: $[-5, -2) \cup (-2, 2) \cup (2, \infty)$

