Answer key and reasoning

D excludes zeros of the original denominator. HA = horizontal asymptote; VA = vertical asymptote; SA = slant asymptote.

1. (x + 3)/(x - 2)

No factor cancels. D: x != 2. VA: x = 2. Holes: none.

2. (x - 1)(x + 4)/[(x - 1)(x + 2)]

Simplifies to (x + 4)/(x + 2), with x != 1, -2.
At x = 1, the limit is 5/3: hole (1, 5/3). VA: x = -2.

3. (x + 2)^2/[(x + 2)(x - 3)]

Simplifies to (x + 2)/(x - 3), with x != -2, 3.
The simplified expression approaches 0 at x = -2: hole (-2, 0). VA: x = 3.

4. (x - 4)/(x - 4)^2

One (x - 4) cancels, but 1/(x - 4) remains. D: x != 4.
VA: x = 4. Holes: none. Partial cancellation is not enough for a hole.

5. (x^2 - 9)/(x^2 + x - 6)

Factor to (x - 3)(x + 3)/[(x + 3)(x - 2)]. D: x != -3, 2.
Simplify to (x - 3)/(x - 2). Hole: (-3, 6/5). VA: x = 2.

6. (x^2 - 4)/(x^2 - 4)

Both factors cancel, leaving 1 only where the original is defined.
D: x != -2, 2. Holes: (-2, 1), (2, 1). VA: none.

Answers for graph features

Intercepts are points on the original graph, so a canceled zero cannot be an x-intercept.

7. (2x - 6)/(x + 1)

D: x != -1; no hole; VA x = -1; HA y = 2 (equal degrees, 2/1).
x-int (3, 0); y-int (0, -6).

8. 4/(x - 2)

D: x != 2; no hole; VA x = 2; HA y = 0 (smaller top degree).
No x-intercept because 4 never equals 0; y-int (0, -2).

9. (x - 3)/(x^2 - 1)

Denominator = (x - 1)(x + 1). D: x != -1, 1; no holes; VAs x = -1, 1; HA y = 0.
x-int (3, 0); y-int (0, 3).

10. (x^2 - 4)/(x^2 + 1)

x^2 + 1 > 0 for real x: D all real; no holes or VAs; HA y = 1.
x-ints (-2, 0), (2, 0); y-int (0, -4).

11. (x + 1)(x - 2)/[(x + 1)(x + 3)]

D: x != -1, -3. Simplify to (x - 2)/(x + 3). Hole (-1, -3/2); VA x = -3; HA y = 1.
x-int (2, 0); y-int (0, -2/3).

12. (x - 2)^2/[(x - 2)(x + 1)]

D: x != 2, -1. Simplify to (x - 2)/(x + 1). Hole (2, 0); VA x = -1; HA y = 1.
No x-intercept: x = 2 is excluded. y-int (0, -2).

Sketch notes and checks

Use the labeled features to check your sketch. Near a vertical asymptote, test one point on either side to place the branches.

13. x/(x - 2)

D: x != 2. Write f = 1 + 2/(x - 2): VA x = 2, HA y = 1.
Both intercepts are (0, 0). Left of x = 2 the curve stays below y = 1 and falls to -infinity; right of x = 2 it stays above y = 1 and rises to +infinity near the pole.

14. (x + 2)/(x^2 - 1)

D: x != -1, 1. VAs x = -1 and x = 1; HA y = 0. x-int (-2, 0); y-int (0, -2).
At -1: left +infinity, right -infinity. At 1: left -infinity, right +infinity. Far left approaches 0 from below; far right from above.

15. (x^2 - 1)/(x - 3)

D: x != 3; VA x = 3. Divide: f = x + 3 + 8/(x - 3), so SA y = x + 3; no HA.
x-ints (-1, 0), (1, 0); y-int (0, 1/3). The curve lies below the slant line to the left of x = 3, above it to the right.

16. (x^2 + x - 2)/(x^2 + 1)

D all real; no holes or VAs; HA y = 1. Factor top: (x + 2)(x - 1).
x-ints (-2, 0), (1, 0); y-int (0, -2). Since f - 1 = (x - 3)/(x^2 + 1), the graph crosses its HA at (3, 1).

Why the factors behave differently

This page makes the distinction between cancellation and domain explicit.

17. Three ratios at x = 2

A = (x - 2)/(x - 2) simplifies to 1 for x != 2, so A has hole (2, 1).
B = (x - 2)^2/(x - 2) simplifies to x - 2 for x != 2, so B has hole (2, 0). That point is not an x-intercept.
C = (x - 2)/(x - 2)^2 simplifies to 1/(x - 2), so C has VA x = 2. All three exclude x = 2.

18. One possible designed function

f(x) = 2(x - 3)(x + 2)/[(x - 1)(x + 2)]. The original domain excludes 1 and -2.
Cancel x + 2 to see VA x = 1, x-intercept (3, 0), and HA y = 2 from the leading-coefficient ratio.
At x = -2, the simplified expression gives 2(-5)/(-3) = 10/3. The hole is (-2, 10/3).

Final check

Keep the original denominator zeros on your page; substitute 0 only when it is allowed; divide the polynomials when you need a slant asymptote.

References: OpenStax, Precalculus 2e, Section 3.7; Common Core HSF.IF.C.7d. The answers were checked using the original denominators and simplified forms. Graphs were sampled on both sides of each vertical asymptote.