Rational Equations: the two list method

Factor, list what is banned, clear the fractions, then check the survivors in the original equation.

THE ROUTINE

1. Factor every denominator.

2. List 1: every value that makes a factor zero.

3. Multiply *every* term by the LCD.

4. Solve. **List 2:** the candidates.

5. Cross out candidates that appear on List 1.

6. Substitute the survivors into the original.

FINDING EXCLUDED VALUES

x excludes 0

$x - 7$ excludes 7

$x^2 - 25$ excludes 5 and -5

$x^2 + 4x$ excludes 0 and -4

$3x - 2$ excludes 2/3

$x^2 - 5x + 6$ excludes 2 and 3

A cancelling factor still excludes. $(x^2 - 4)/(x - 2)$ equals $x + 2$ everywhere except $x = 2$, where the graph has a hole. Take the list from the denominator before you cancel.

WHICH CLEARING METHOD

Cross multiply only when the equation is one fraction equal to one fraction.

Multiply by the LCD in every other case, including when a whole number or a variable term sits outside a fraction.

WHY THE CHECK IS REQUIRED

Clearing denominators gives an equation that is equivalent to the original one only for the values the original allows.

The polynomial you are left with is defined everywhere, so it can carry roots at excluded values. Those roots have to be rejected.

THREE WAYS AN ANSWER CAN END

Solutions: one or more candidates survive.

No solution: every candidate is excluded.

Identity: the cleared equation reduces to a true statement, so every value works except the excluded ones.

EXTRANEOUS OR NON VIABLE

Extraneous: makes a denominator zero, so it breaks the equation. Reject on algebra.

Non viable: satisfies the equation but contradicts the situation, such as a negative time or length. Reject on the context, and say which.

SETTING UP THE WORD PROBLEMS

Work together: 1/a + 1/b = 1/(hours together). Each fraction is the share of the job done in one hour.

Equal times: d1/r1 = d2/r2. With a current or a wind the rates are r + *c* and r - *c*.

Reaching a percent: (part + *n*)/(whole + *n*) = target.

BEFORE YOU HAND IT IN

List 1 written down, from the *factored* denominators?

Every term multiplied by the LCD, whole numbers included?

Brackets on each subtracted numerator?

Every survivor checked in the *original* equation?

Nothing survived? Write "no solution", do not leave it blank.

ONE PROBLEM, START TO FINISH

$$\frac{x}{x - 4} + \frac{1}{x + 1} = \frac{20}{x^2 - 3x - 4}$$

Factor the last denominator: $(x - 4)(x + 1)$. **List 1:** 4 and -1. Multiply every term by the LCD: $x(x + 1) + (x - 4) = 20$, so $x^2 + 2x - 24 = 0$ and $(x + 6)(x - 4) = 0$. **List 2:** -6 and 4.

The candidate 4 is on List 1, so it is extraneous. Substitute -6 into the original equation: both sides equal 2/5. **Solution: $x = -6$**

THE FOUR TRAPS

Solving before writing List 1	Write List 1 in the margin, boxed, first.
Forgetting to multiply a whole number by the LCD	Write the multiplication above every term.
Distributing a subtraction without brackets	Bracket the second numerator before distributing.
Checking the candidate in the cleared equation	Check in the equation that still has denominators.

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