Multiplying and Dividing Rational Expressions

Algebra 2 practice · 24 problems · answer key on pages 8 to 10

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Name _____ Date _____ Period _____

BEFORE YOU START

Factor every numerator and denominator first. Then write the restrictions on the variable, before you flip or cancel anything. Set each denominator equal to zero, and in a division problem set the numerator of the divisor equal to zero as well. A restriction does not disappear when its factor cancels, so the list you write in step two is the list you report in your answer.

Part A. Multiplying, one factoring step or none

Simplify completely and state every restriction on the variable.

A1. $\dfrac{6x^3}{5y^2} \cdot \dfrac{10y^4}{9x}$

A2. $\dfrac{x}{x+3} \cdot \dfrac{x+3}{x-2}$

A3. $\dfrac{x+5}{2x} \cdot \dfrac{8x^2}{x+5}$

A4. $\dfrac{3x-9}{x^2} \cdot \dfrac{x}{x-3}$

A5. $\dfrac{x^2-16}{x+2} \cdot \dfrac{x+2}{x-4}$

A6. $\dfrac{4x+8}{3x-6} \cdot \dfrac{x-2}{x+2}$

Part B. Multiplying with full factoring

Factor each numerator and denominator before you cancel anything.

B1. $\dfrac{x^2 + 7x + 12}{x^2 - 9} \cdot \dfrac{x^2 - 3x}{x + 4}$

B2. $\dfrac{x^2 - 5x + 6}{x^2 - 4} \cdot \dfrac{x^2 + 6x + 8}{x^2 - 9}$

B3. $\dfrac{2x^2 + 5x - 3}{x^2 - 1} \cdot \dfrac{x^2 + 2x + 1}{2x - 1}$

Part B. Multiplying with full factoring, continued

B4. $\dfrac{3x^2 - 12}{x^2 + 4x + 4} \cdot \dfrac{x + 2}{6x - 12}$

B5. $\dfrac{x^2 - x - 20}{x^2 + 2x - 15} \cdot \dfrac{x^2 - 9}{x^2 - 16}$

B6. $\dfrac{4x^2 - 9}{2x^2 + x - 6} \cdot \dfrac{x^2 + 5x + 6}{2x + 3}$

Part C. Dividing

Multiply by the reciprocal. The numerator of the divisor cannot be zero either.

C1. $\dfrac{x^2}{3y} \div \dfrac{x}{9y^3}$

C2. $\dfrac{x+4}{x-1} \div \dfrac{x+4}{x+1}$

C3. $\dfrac{x^2-25}{x+1} \div \dfrac{x-5}{3x+3}$

Part C. Dividing, continued

C4. $\dfrac{6x^2 - 6}{x^2 + 3x} \div \dfrac{x + 1}{x + 3}$

C5. $\dfrac{x^2 + 2x - 8}{x^2 - x - 6} \div \dfrac{x^2 + 5x + 4}{x^2 - 2x - 3}$

C6. $\dfrac{3x^2 + 11x + 6}{x^2 - 4} \div \dfrac{3x + 2}{x^2 - 4x + 4}$

Part D. Mixed, with opposite binomials and cubics

Watch for binomials written backwards and for factoring by grouping.

D1. $\dfrac{x^2 - 9}{x^2 + 6x + 9} \div \dfrac{x^2 - 3x}{x^2 + 3x}$

D2. $\dfrac{x^2 - 2x - 15}{x^2 - 25} \cdot \dfrac{x^2 + 10x + 25}{x^2 + 2x - 3} \div \dfrac{x + 5}{x - 2}$

D3. $\dfrac{x^2 - 4}{6 - 3x} \cdot \dfrac{9x}{x + 2}$

Part D. Mixed, with opposite binomials and cubics, continued

D4. $\dfrac{x^3 - 8}{x^2 + 2x + 4} \div \dfrac{x^2 - 4}{x^2 + 4x + 4}$

D5. $\dfrac{x^3 + 3x^2 - 4x - 12}{x^2 - 4} \cdot \dfrac{x - 2}{x + 3}$

D6. $\dfrac{x^2 - 1}{x^2 + 5x + 6} \div \dfrac{x^2 + 2x + 1}{x^2 - 9}$

Every answer is given with its complete restriction list, including restrictions whose factors cancelled.

A1. $\frac{4x^2y^2}{3}$ where $x \neq 0,\ y \neq 0$

Multiply across: $60x^3y^4$ over $45xy^2$, then reduce.

A2. $\frac{x}{x-2}$ where $x \neq -3,\ x \neq 2$

$(x + 3)$ cancels but its restriction stays.

A3. $4x$ where $x \neq 0,\ x \neq -5$

$8x^2$ over $2x$ reduces to $4x$.

A4. $\frac{3}{x}$ where $x \neq 0,\ x \neq 3$

Factor $3x - 9$ as $3(x - 3)$ first.

A5. $x + 4$ where $x \neq -2,\ x \neq 4$

$x^2 - 16 = (x - 4)(x + 4)$.

A6. $\frac{4}{3}$ where $x \neq 2,\ x \neq -2$

Every variable factor cancels. The value is the constant 4/3 wherever it is defined.

B1. x where $x \neq 3,\ x \neq -3,\ x \neq -4$

$(x + 3)(x + 4)$ over $(x - 3)(x + 3)$ times $x(x - 3)$ over $(x + 4)$. All three restrictions vanish from the answer.

B2. $\frac{x+4}{x+3}$ where $x \neq 2,\ x \neq -2,\ x \neq 3,\ x \neq -3$

$(x - 2)(x - 3)$ over $(x - 2)(x + 2)$ times $(x + 2)(x + 4)$ over $(x - 3)(x + 3)$.

B3. $\dfrac{(x+1)(x+3)}{x-1}$ where $x \neq 1,\ x \neq -1,\ x \neq 1/2$

$2x^2 + 5x - 3 = (2x - 1)(x + 3)$. The $(2x - 1)$ gives the fractional restriction.

B4. $\dfrac{1}{2}$ where $x \neq -2,\ x \neq 2$

$3(x - 2)(x + 2)$ over $(x + 2)^2$ times $(x + 2)$ over $6(x - 2)$. Only 3/6 survives.

B5. $\dfrac{(x-5)(x+3)}{(x+5)(x-4)}$ where $x \neq -5,\ x \neq 3,\ x \neq 4,\ x \neq -4$

$(x - 5)(x + 4)$ over $(x + 5)(x - 3)$ times $(x - 3)(x + 3)$ over $(x - 4)(x + 4)$.

B6. $x + 3$ where $x \neq 3/2,\ x \neq -2,\ x \neq -3/2$

$2x^2 + x - 6 = (2x - 3)(x + 2)$, and $4x^2 - 9 = (2x - 3)(2x + 3)$.

C1. $3xy^2$ where $x \neq 0,\ y \neq 0$

The $x \neq 0$ comes from the divisor numerator, not from any denominator you can see.

C2. $\dfrac{x+1}{x-1}$ where $x \neq 1,\ x \neq -1,\ x \neq -4$

The divisor numerator $(x + 4)$ gives $x \neq -4$.

C3. $3(x + 5)$ where $x \neq -1,\ x \neq 5$

Flip first: $(x - 5)(x + 5)$ over $(x + 1)$ times $3(x + 1)$ over $(x - 5)$.

C4. $\dfrac{6(x-1)}{x}$ where $x \neq 0,\ x \neq -3,\ x \neq -1$

$6(x - 1)(x + 1)$ over $x(x + 3)$ times $(x + 3)$ over $(x + 1)$.

C5. $\dfrac{x-2}{x+2}$ where $x \neq 3,\ x \neq -2,\ x \neq -4,\ x \neq -1$

Two of the four restrictions come from the divisor numerator $(x + 4)(x + 1)$.

C6. $\dfrac{(x-2)(x+3)}{x+2}$ where $x \neq 2,\ x \neq -2,\ x \neq -2/3$

$3x^2 + 11x + 6 = (3x + 2)(x + 3)$, and $x^2 - 4x + 4 = (x - 2)^2$.

D1. 1 where $x \neq -3,\ x \neq 0,\ x \neq 3$

The value is 1 at every number where it is defined, and it is undefined at three of them.

D2. $\dfrac{x-2}{x-1}$ where $x \neq 5,\ x \neq -5,\ x \neq 1,\ x \neq -3,\ x \neq 2$

Work left to right. The divisor numerator $(x + 5)$ contributes $x \neq -5$.

D3. $-3x$ where $x \neq 2,\ x \neq -2$

$6 - 3x = -3(x - 2)$. Missing the negative gives the answer the wrong sign.

D4. $x + 2$ where $x \neq 2,\ x \neq -2$

$x^3 - 8 = (x - 2)(x^2 + 2x + 4)$. The quadratic factor is never zero for a real number, so it adds no restriction.

D5. $x - 2$ where $x \neq 2,\ x \neq -2,\ x \neq -3$

Group: $x^2(x + 3) - 4(x + 3) = (x + 3)(x - 2)(x + 2)$.

D6. $\dfrac{(x-1)(x-3)}{(x+1)(x+2)}$ where $x \neq -2,\ x \neq -3,\ x \neq -1,\ x \neq 3$

$x \neq -1$ comes from the divisor numerator $(x + 1)^2$.