Multiplying and Dividing Rational Expressions

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THE RULES

$$\frac{A}{B} \cdot \frac{C}{D} = \frac{AC}{BD}$$

$$\frac{A}{B} \div \frac{C}{D} = \frac{A}{B} \cdot \frac{D}{C}$$

Flip the second fraction only. Never the first, never both.

FIVE STEPS, IN THIS ORDER

1. Factor every numerator and denominator.
2. Write the restrictions.
3. Flip if dividing, then cancel factors.
4. Multiply what is left across.
5. Report the answer with the step 2 list.

WHERE RESTRICTIONS COME FROM

Source	Multiply	Divide
Denominator of the first fraction	Yes	Yes
Denominator of the second fraction	Yes	Yes
Numerator of the second fraction	No	Yes, it is the divisor
Numerator of the first fraction	No	No

Restrictions never disappear when factors cancel. A cancelled factor leaves a hole in the graph, not a clean domain. Write the list from the original expression, before flipping or cancelling.

FACTORING CHECKLIST

- GCF first, every time
- $a^2 - b^2 = (a - b)(a + b)$
- $x^2 + bx + c$: two numbers, product c, sum b
- $ax^2 + bx + c$: split the middle term
- Four terms: group in pairs
- $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

OPPOSITE BINOMIALS

$$\frac{a - x}{x - a} = -1$$

Factor -1 out of the binomial written backwards, then the factors match. Circle any binomial where the variable comes second. Missing it gives the answer the wrong sign.

WHAT YOU MAY NOT CANCEL

Legal: a factor, which is multiplied by everything else. $(x + 3)(x - 1) \div (x + 3) = x - 1$

Not legal: a term, which is added. $(x + 3)/3$ does not become x, and $(x^2 + 4)/x^2$ does not become 4.

Read it out loud. If you say the word “plus” between the piece you want to cancel and its neighbor, it stays.

ONE WORKED DIVISION EXAMPLE

$$\frac{x^2 - 9}{x^2 + 2x} \div \frac{x - 3}{x} = \frac{(x - 3)(x + 3)}{x(x + 2)} \cdot \frac{x}{x - 3} = \frac{x + 3}{x + 2}$$

Restrictions: $x \neq 0$ and $x \neq -2$ from the first denominator, and $x \neq 3$ from the divisor numerator. Two of those three cannot be read off the simplified answer.

SELF CHECK BEFORE YOU HAND IT IN

Pick a value of x that is not on your restriction list, usually $x = 1$ or $x = 10$. Evaluate the original expression and your simplified answer. If the two numbers disagree, the error is somewhere between them, and you can find it before you hand in the page.