Graphing Rational Functions

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THE SIX STEPS

1. Factor the numerator and the denominator completely.
2. **Excluded values** from the **original** denominator.
3. Cancel, then read the **reduced** denominator. Its zeros are the **vertical asymptotes**. Any other excluded value is a **hole**.
4. **Intercepts.** Reduced numerator = 0 for the x-intercepts, discarding excluded values. Evaluate at x = 0 for the y-intercept.
5. **End behavior** by comparing degrees in the reduced form.
6. **Sign chart.** Cut at every real zero of the reduced numerator and the reduced denominator, then test one allowed value per interval.

HOLE OR ASYMPTOTE

Factor cancels completely	**Hole.** Height from the reduced function.
A copy remains in the reduced denominator	**Vertical asymptote,** even though something cancelled.
Nothing cancels	**Vertical asymptote.**

Read the reduced denominator. Cancelling alone does not decide it.

DEGREE TEST, USING THE REDUCED FORM

n < d	Horizontal asymptote y = 0
n = d	Horizontal asymptote y = ratio of leading coefficients
n = d + 1	Slant asymptote: divide, keep the quotient, drop the remainder
n > d + 1	Neither. The graph follows the polynomial quotient

BRANCH DIRECTION

Substitute a number just left of the asymptote and again just right, into the **whole reduced function**. Positive means that side runs up, negative means it runs down. The other denominator factors carry signs, so the numerator alone is not enough.

WHERE MARKS GET LOST

Reading the vertical asymptotes off the **original** denominator, which turns every hole into an asymptote.

Assuming a cancelling factor always gives a hole. If a copy remains below, it is still an asymptote.

Naming a hole without its y-coordinate.

Testing the sign with the numerator alone instead of the whole reduced function.

Treating a horizontal asymptote as a barrier the graph cannot cross.

Cutting the sign chart at the x-intercepts rather than at the zeros of the reduced numerator.

THE SEVEN FEATURES, AND WHERE EACH ONE COMES FROM

Excluded values	Zeros of the **original** denominator
Holes	Factors that cancel completely
Vertical asymptotes	Zeros of the **reduced** denominator
x-intercepts	Zeros of the reduced numerator the domain allows
y-intercept	Evaluate at x = 0, if 0 is allowed
End behavior	Degree of the numerator against the denominator
Sign on each interval	One allowed test value per interval

DOES THE GRAPH MEET THAT ASYMPTOTE

Solve **N(x) - k · D(x) = 0** with the reduced N and D, then discard any excluded root. For a slant asymptote, put the linear quotient in place of k.

Odd multiplicity: the graph crosses and continues to the other side.

Even multiplicity: the graph touches and returns to the same side.

No real roots: the graph never reaches it.

Everything cancels: the graph lies on that line wherever it is defined.

SIGN CHART RULES

Cut at the zeros of the reduced **numerator** and reduced **denominator**, not at the x-intercepts.

No test value may be an excluded x.

Even multiplicity means **no** sign change at that cut point.

Intervals are not branches. Count branches from the vertical asymptotes.

THREE ANSWERS THAT LOOK WRONG AND ARE NOT

A function with a hole and no asymptote at all, when the denominator cancels away entirely.

A graph that crosses its own horizontal asymptote, once or several times.

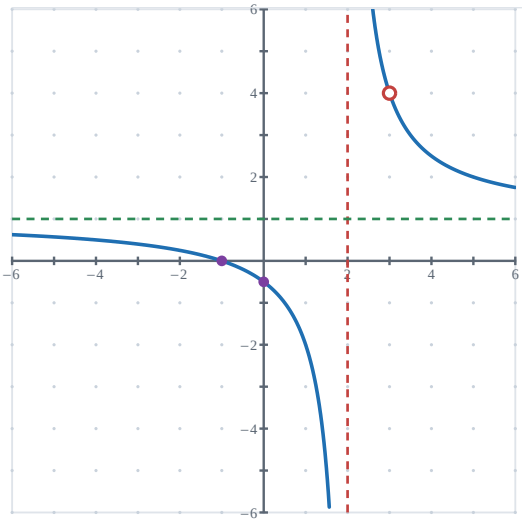
A sign chart cut point where the sign does not change.

WORKED IN ONE LINE

$$\frac{x^2-2x-3}{x^2-5x+6} = \frac{(x-3)(x+1)}{(x-3)(x-2)} \rightarrow \frac{x+1}{x-2}$$

Excluded 3 and 2. Reduced denominator is (x - 2), so the vertical asymptote is **x = 2** and x = 3 is a **hole at (3, 4)**. x-intercept **(-1, 0)**, y-intercept **(0, -1/2)**, horizontal asymptote **y = 1**, signs **+ - +**.

WHAT THE FINISHED GRAPH LOOKS LIKE



Blue the function

Red dashed vertical asymptote x = 2

Red circle hole at (3, 4)

Green dashed horizontal asymptote y = 1

Purple intercepts (-1, 0) and (0, -1/2)

Signs + - + across the cut points -1 and 2

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