Complex Rational Expressions: Reference Card

Everything needed to simplify a fraction that has fractions inside it, on one page.

METHOD A: COMBINE, THEN DIVIDE

1. Write the top level as one fraction.
2. Write the bottom level as one fraction.
3. Multiply the top by the reciprocal of the bottom.
4. Factor everything, then cancel common factors.

METHOD B: MULTIPLY BY THE LCD

1. Factor every inner denominator.
2. Build the LCD of all inner fractions, both levels.
3. Multiply every term on both levels by that LCD.
4. Factor the result, then cancel common factors.

THE THREE RESTRICTION SOURCES

- **1.** Every inner denominator in the top level is nonzero.
- **2.** Every inner denominator in the bottom level is nonzero.
- **3.** The whole bottom level is nonzero. Combine or clear the bottom, set the result equal to zero, and keep the solutions that the inner denominators allow.

RULES THAT DECIDE THE ANSWER

- Start the list with the inner-denominator restrictions, then add the lower-expression ones as soon as you combine or clear the bottom.
- Cancel factors only. Never cancel across a plus or minus sign.
- Rewrite every negative exponent as a fraction first.
- When an entire expression is raised to a negative power, its base becomes a denominator and must be nonzero.

WORKED IN ONE LINE

$$\frac{\dfrac{1}{x}}{\dfrac{1}{x}-\dfrac{1}{2}} = \frac{2}{2-x} \qquad x \neq 0,\ 2$$

Sources 1 and 2 give $x \neq 0$ from the inner denominator. Source 3 gives $x \neq 2$, because the bottom level is zero there. The simplified form shows only $x \neq 2$, so the other restriction has to be carried down from the first line.

Choosing a method

Method A is usually convenient when each level is already one fraction or can be combined quickly. Method B is often shorter when several fractional terms share a manageable LCD. Either way, find the Source 3 restriction before writing the final answer.

Two more examples

$$\frac{1-\dfrac{4}{x^2}}{1+\dfrac{2}{x}} = \frac{x-2}{x} \qquad x \neq -2,\ 0$$

$x \neq -2$ comes from the bottom level and leaves no trace in the answer.

$$(a^{-1}+b^{-1})^{-1} = \frac{ab}{a+b} \qquad a \neq 0,\ b \neq 0,\ a+b \neq 0$$

An outer negative exponent is a division, so the inner sum must be nonzero.

Five mistakes to check for

1. Rewriting the complex fraction on one line without parentheses around each level.
2. Reading the restrictions off the simplified answer instead of the original expression.
3. Skipping Source 3, the restriction that comes from the whole bottom level being zero.
4. Multiplying only the fractional terms by the LCD and leaving the constants behind.
5. Inverting the top fraction instead of the bottom one.

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