

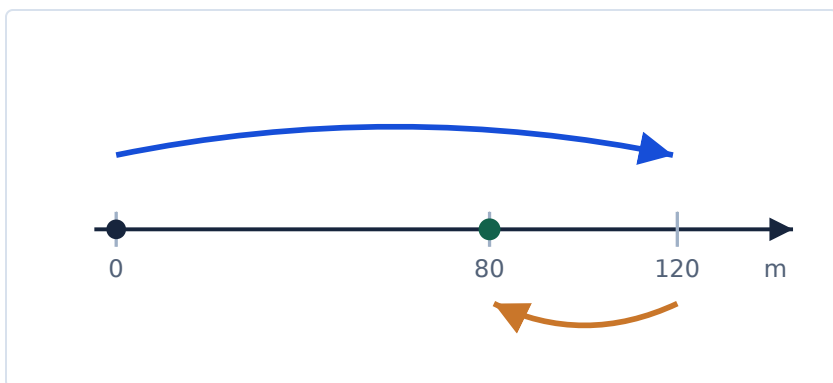
Kinematics Answer Key

Every solution shows the reasoning, not just the final number. Intermediate values are carried unrounded and reported to two significant figures at the end, since $g = 9.8 \text{ m/s}^2$ carries only two.

Part A: Reading motion and graphs

Problems 1 to 8

1. A runner jogs 120 m east in 30 s, turns around, and jogs 40 m west in 10 s. Taking east as positive, find the displacement, the distance, the average velocity, and the average speed for the whole trip. The trip is sketched below.



- Left dot**
Start, at the origin
- Upper arrow**
First leg, 30 s
- Lower arrow**
Second leg, 10 s
- Green dot**
Finish

Take east as positive.

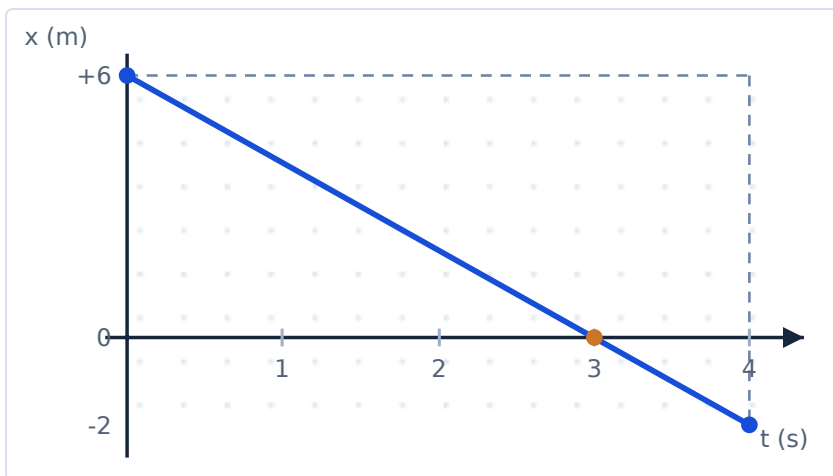
$\Delta x = +120 - 40 = +80 \text{ m}$. Distance = $120 + 40 = 160 \text{ m}$.

Total time = $30 + 10 = 40 \text{ s}$.

$v_{\text{avg}} = +80 / 40 = +2.0 \text{ m/s}$ $\text{speed}_{\text{avg}} = 160 / 40 = 4.0 \text{ m/s}$

Displacement and distance answer different questions about the same trip.

2. Use the graph below. Find the velocity and the displacement over the interval shown.



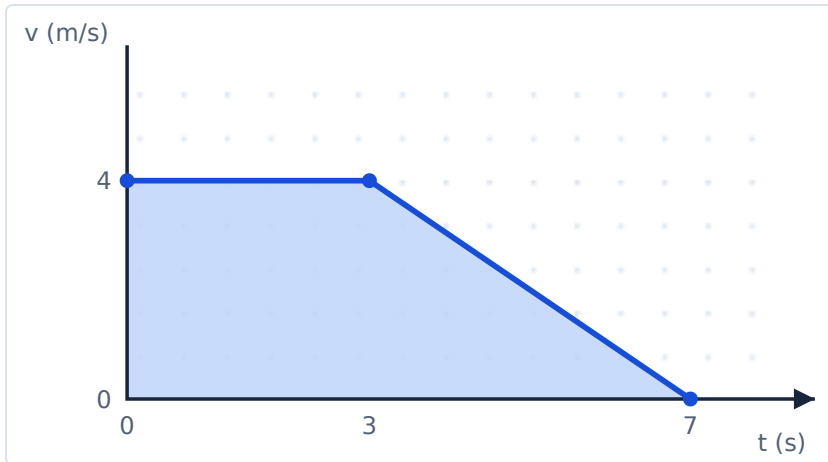
- Blue line**
Position, +6.0 m to -2.0 m
- Orange point**
Passes the origin
- Dashed guides**
Read the endpoint values
- Slope**
The quantity you want first

$v = \Delta x / \Delta t = (-2.0 - 6.0) / 4.0 = -2.0 \text{ m/s}$

$\Delta x = -2.0 - 6.0 = -8.0 \text{ m}$

On an x vs t graph the slope is the velocity. The negative sign means motion in the negative direction, not deceleration.

3. Use the graph below. Find the total displacement over the full 7.0 s and the acceleration during the second phase.



Blue line

Velocity

Shaded region

Area = displacement

First phase

Constant, 0 to 3.0 s

Second phase

Straight to zero at 7.0 s

Phase 1 area: $(4.0 \text{ m/s})(3.0 \text{ s}) = 12 \text{ m}$

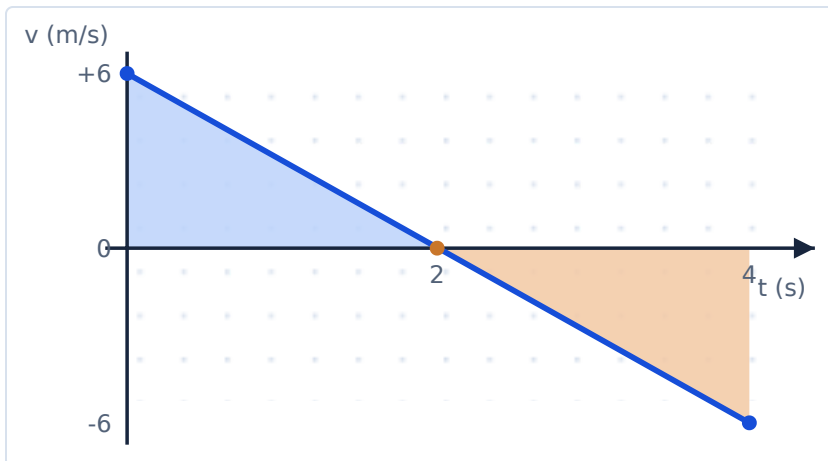
Phase 2 area: $\frac{1}{2}(4.0 \text{ s})(4.0 \text{ m/s}) = 8.0 \text{ m}$

$\Delta x = 12 + 8.0 = +20 \text{ m}$

$a_2 = (0 - 4.0) / (7.0 - 3.0) = -1.0 \text{ m/s}^2$

The object never reverses, so the distance is also 20 m.

4. Use the graph below. Find the acceleration, the displacement, and the distance traveled over the 4.0 s.



Blue line

Velocity, +6.0 to -6.0 m/s

Orange point

Velocity reverses here

Upper region

Positive area

Lower region

Negative area

$a = (-6.0 - 6.0) / 4.0 = -3.0 \text{ m/s}^2$

Velocity crosses zero at $t = 2.0 \text{ s}$.

Area above: $\frac{1}{2}(2.0)(6.0) = +6.0 \text{ m}$ Area below: $\frac{1}{2}(2.0)(6.0) = -6.0 \text{ m}$

Displacement = **0** Distance = $6.0 + 6.0 = 12 \text{ m}$

This is the classic case where displacement and distance disagree. Read the prompt carefully.

5. An object has negative velocity and positive acceleration. Is it speeding up or slowing down? Explain in one sentence.

Slowing down.

Velocity and acceleration have opposite signs, so the acceleration opposes the motion and the speed decreases toward zero.

6. An object starts from rest and has a constant acceleration of $+2.0 \text{ m/s}^2$ for 5.0 s . Find its final velocity and its displacement.

$$v = v_0 + at = 0 + (2.0)(5.0) = \mathbf{+10 \text{ m/s}}$$

$$\Delta x = \frac{1}{2}at^2 = \frac{1}{2}(2.0)(5.0)^2 = \mathbf{+25 \text{ m}}$$

$$\text{Check: } \Delta x = \frac{1}{2}(0 + 10)(5.0) = 25 \text{ m.}$$

7. A motion diagram shows dots every 0.10 s with gaps of 2.0 , 4.0 , 6.0 , and 8.0 cm . Is the acceleration constant? If so, what is it?

Average velocity in each interval = gap / 0.10 s : 0.20 , 0.40 , 0.60 , 0.80 m/s .

Each rises by 0.20 m/s per 0.10 s , so $a = 0.20 / 0.10 = \mathbf{+2.0 \text{ m/s}^2}$, **constant**

Equal increases in the gaps are the signature of constant acceleration.

8. An object starts from rest under constant acceleration. Measurements give $\Delta x = 0.75 \text{ m}$ at 1.0 s , 3.0 m at 2.0 s , and 6.75 m at 3.0 s . Find the slope of Δx against t^2 , then find the acceleration.

$\Delta x / t^2 = 0.75 / 1.0 = 3.0 / 4.0 = 6.75 / 9.0 = 0.75 \text{ m/s}^2$ for all three points, so the plot is linear with **slope 0.75 m/s^2** .

Slope = $a/2$, so $a = \mathbf{1.5 \text{ m/s}^2}$

Linearizing data this way is a standard experimental design task on the exam.

Part B: Constant acceleration in one dimension

Problems 9 to 16

9. A car starts from rest and accelerates at 3.0 m/s^2 for 5.0 s . Find its final speed and the distance covered.

$$v = 0 + (3.0)(5.0) = \mathbf{15 \text{ m/s}}$$

$$\Delta x = \frac{1}{2}(3.0)(5.0)^2 = \mathbf{37.5 \text{ m}}$$

Both equations were chosen because they contain only known quantities plus the target.

10. A car traveling at 25 m/s brakes uniformly to a stop in 40 m . Find the acceleration.

$$0 = (25)^2 + 2a(40)$$

$$a = -625 / 80 = \mathbf{-7.8 \text{ m/s}^2}$$

$v^2 = v_0^2 + 2a\Delta x$ is the right choice because time is neither known nor asked for.

11. For the car in problem 10, how long does the stop take?

$$t = (v - v_0) / a = (0 - 25) / (-7.8125) = \mathbf{3.2 \text{ s}}$$

Use the unrounded acceleration to avoid compounding rounding error.

12. An object is launched along a track at $+8.0 \text{ m/s}$ with a constant acceleration of -2.0 m/s^2 . When does it return to its starting point, and what is its velocity then?

$$\text{Set } \Delta x = 0: 0 = (8.0)t - \frac{1}{2}(2.0)t^2 = t(8.0 - t)$$

$$t = 0 \text{ (the start) or } \mathbf{t = 8.0 \text{ s}}$$

$$v = 8.0 + (-2.0)(8.0) = \mathbf{-8.0 \text{ m/s}}$$

It comes back at the same speed in the opposite direction, which is the symmetry of constant acceleration.

13. A cyclist accelerates from rest at 2.0 m/s^2 for 6.0 s , then holds that speed for 10 s . Find the total distance.

$$\text{Stage 1: } v = (2.0)(6.0) = 12 \text{ m/s, } \Delta x = \frac{1}{2}(2.0)(6.0)^2 = 36 \text{ m}$$

$$\text{Stage 2: } \Delta x = (12)(10) = 120 \text{ m}$$

$$\text{Total} = \mathbf{156 \text{ m}}$$

The end conditions of stage 1 become the start conditions of stage 2.

14. A train moving at 30 m/s decelerates at 1.2 m/s^2 . How far does it travel in the first 5.0 s , and how far during the fifth second alone?

$$\Delta x(5.0) = (30)(5.0) - \frac{1}{2}(1.2)(5.0)^2 = 150 - 15 = \mathbf{135 \text{ m}}$$

$$\Delta x(4.0) = (30)(4.0) - \frac{1}{2}(1.2)(4.0)^2 = 120 - 9.6 = 110.4 \text{ m}$$

$$\text{Fifth second} = 135 - 110.4 = \mathbf{24.6 \text{ m}}$$

Distance during one particular second is always a difference of two cumulative displacements.

15. Object A starts from rest at the origin with $a = 2.0 \text{ m/s}^2$. Object B passes the origin at the same instant moving at a constant 6.0 m/s . When and where does A catch B?

$$\text{A: } x = \frac{1}{2}(2.0)t^2 = t^2 \quad \text{B: } x = 6.0t$$

$$t^2 = 6.0t, \text{ so } t = 0 \text{ or } \mathbf{t = 6.0 \text{ s}}$$

$$x = (6.0)(6.0) = \mathbf{36 \text{ m}}$$

Setting two position expressions equal is the standard catch-up method. Discard $t = 0$, the moment they were together at the start.

16. A rocket sled accelerates uniformly from 50 m/s to 150 m/s over 500 m. Find the acceleration and the elapsed time.

$$a = (150^2 - 50^2) / (2 \times 500) = 20000 / 1000 = \mathbf{20 \text{ m/s}^2}$$

$$t = (150 - 50) / 20 = \mathbf{5.0 \text{ s}}$$

No time in the first equation, no displacement in the second. Each step avoids an unnecessary unknown.

Part C: Free fall and projectiles

Problems 17 to 24

17. A ball is dropped from rest from a height of 20 m. Find the time to fall and the speed on impact.

$$t = \sqrt{2h/g} = \sqrt{2 \times 20 / 9.8} = \mathbf{2.0 \text{ s}}$$

$$v = gt = (9.8)(2.02) = \mathbf{20 \text{ m/s}}$$

Or use $v^2 = 2g h$ directly: $v = \sqrt{2 \times 9.8 \times 20} = 19.8 \text{ m/s}$.

18. A ball is thrown straight up at 15 m/s. Find the time to reach the top, the maximum height above the launch point, and the total time to return to launch height.

$$t_{\text{up}} = v_0/g = 15 / 9.8 = \mathbf{1.5 \text{ s}}$$

$$H = v_0^2 / 2g = 225 / 19.6 = \mathbf{11 \text{ m}}$$

$$T = 2t_{\text{up}} = \mathbf{3.1 \text{ s}}$$

The trip up and the trip down take the same time only because it returns to the launch height.

19. A ball is thrown straight up at 12 m/s from a hand 1.5 m above the ground. Find its speed when it hits the ground and the total time in the air.

Up positive, origin at the hand, so $\Delta y = -1.5 \text{ m}$ at the ground.

$$v^2 = (12)^2 + 2(9.8)(1.5) = 173.4, v = \mathbf{13 \text{ m/s downward}}$$

$$t = (12 + 13.17) / 9.8 = \mathbf{2.6 \text{ s}}$$

The launch and landing heights differ, so $T = 2v_0/g$ would give the wrong answer here.

20. A stone is thrown horizontally at 20 m/s from a 45 m cliff. Find the time in the air, the horizontal distance, and the impact speed.

$$\text{Vertical: } t = \sqrt{2 \times 45 / 9.8} = \mathbf{3.0 \text{ s}}$$

$$\text{Horizontal: } \Delta x = (20)(3.03) = \mathbf{61 \text{ m}}$$

$$v_y = (9.8)(3.03) = 29.7 \text{ m/s, so } v = \sqrt{20^2 + 29.7^2} = \mathbf{36 \text{ m/s}}$$

Impact speed combines both components. The horizontal one never changed.

21. A projectile is launched at 25 m/s at 40° above the horizontal from level ground. Find the time of flight, the range, and the maximum height.

$$v_{0x} = 25\cos 40^\circ = 19.2 \text{ m/s} \quad v_{0y} = 25\sin 40^\circ = 16.1 \text{ m/s}$$

$$T = 2(16.1) / 9.8 = \mathbf{3.3 \text{ s}}$$

$$R = (19.2)(3.28) = \mathbf{63 \text{ m}}$$

$$H = (16.1)^2 / 19.6 = \mathbf{13 \text{ m}}$$

Level ground, so the shortcuts apply. Check against $R = v_0^2 \sin(2\theta)/g = 625 \sin 80^\circ / 9.8 = 63 \text{ m}$.

22. Two projectiles are launched at 25 m/s from level ground, one at 30° and one at 60° . Compare their ranges and their maximum heights, with numbers.

$$R = v_0^2 \sin(2\theta)/g. \sin 60^\circ = \sin 120^\circ, \text{ so both ranges are } \mathbf{55 \text{ m}}.$$

$$H_{30} = (12.5)^2 / 19.6 = \mathbf{8.0 \text{ m}} \quad H_{60} = (21.7)^2 / 19.6 = \mathbf{24 \text{ m}}$$

Complementary angles always give equal ranges on level ground. Everything else about the two flights differs.

23. A plane flying horizontally at 60 m/s at an altitude of 500 m releases a package. Find the time to fall and the horizontal distance. Where is the plane when the package lands, assuming it holds its velocity?

$$t = \sqrt{2 \times 500 / 9.8} = \mathbf{10 \text{ s}}$$

$$\Delta x = (60)(10.1) = \mathbf{610 \text{ m}}$$

The plane is **directly above the package**.

Both keep the same horizontal velocity and neither has horizontal acceleration, so they stay vertically aligned.

24. From a 3.0 m height, one ball is dropped from rest and another is launched horizontally at 5.0 m/s. Which lands first, and how far from the base does the second one land?

Both have $v_{0y} = 0$ and the same drop, so both take $t = \sqrt{2 \times 3.0 / 9.8} = \mathbf{0.78 \text{ s}}$. **They land at the same time.**

$$\Delta x = (5.0)(0.78) = \mathbf{3.9 \text{ m}}$$

Horizontal velocity has no effect on fall time. This is the single most useful idea in projectile motion.