Adding and Subtracting Rational Expressions

20 problems in four parts, with a complete answer key. Written by Berke Sahbazoglu, Burke Tutoring in Fremont, CA. Free to print and use in class.

Name _____ Date _____ Period _____

Directions. Simplify each expression completely and state every restriction on the variable. Take the restrictions from the original denominators, before any rewriting, and keep them even when the factor that caused them cancels. Leave denominators in factored form.

Part A. Denominators that already match

A1 $\dfrac{3x}{x+4} + \dfrac{12}{x+4}$

A2 $\dfrac{5x-1}{x-2} - \dfrac{3x+7}{x-2}$

A3 $\dfrac{x^2}{x-5} - \dfrac{25}{x-5}$

A4 $\dfrac{4x+3}{x^2-1} + \dfrac{2x-9}{x^2-1}$

A5 $\dfrac{x+7}{2x+6} - \dfrac{2x+1}{2x+6}$

Part B. Unlike denominators, one build up step each

B1 $\frac{3}{x} + \frac{5}{x+1}$

B2 $\frac{2}{x-3} + \frac{4}{x+1}$

B3 $\frac{5}{x+2} - \frac{3}{x-4}$

B4 $\frac{x}{x+5} + \frac{3}{x-1}$

B5 $\frac{7}{2x} - \frac{3}{x+4}$

Part C. Factor the denominators first

C1 $\dfrac{4}{x^2 - 9} + \dfrac{2}{x + 3}$

C2 $\dfrac{3}{x^2 + 5x + 6} - \dfrac{2}{x + 2}$

C3 $\dfrac{5}{x^2 - 4} - \dfrac{3}{x^2 - 4x + 4}$

C4 $\dfrac{2x}{x^2 - x - 6} + \dfrac{1}{x + 2}$

C5 $\dfrac{x + 1}{x^2 - 5x + 6} - \dfrac{x - 1}{x^2 - 4}$

Part D. Opposite denominators, and answers that reduce

D1 $\dfrac{x}{x-7} + \dfrac{7}{7-x}$

D2 $\dfrac{3}{x-2} - \dfrac{5}{2-x}$

D3 $\dfrac{x}{x^2-9} + \dfrac{3}{3-x}$

D4 $\dfrac{x+5}{x^2-1} - \dfrac{3}{x-1}$

D5 $\dfrac{2x}{x^2-16} - \dfrac{1}{x-4}$

Challenge set

E1 $\dfrac{2}{x-1} + \dfrac{3}{x+1} - \dfrac{4}{x^2-1}$

E2 $\dfrac{x}{x^2+2x+1} - \dfrac{1}{x+1}$

E3 $\dfrac{5}{x-3} - \dfrac{2}{3-x} + \dfrac{1}{x+3}$

Answer key, Parts A and B

Each answer shows the LCD used, the numerator before simplifying, the final expression, and the full restriction list. Questions were checked for accuracy before publishing.

Part A

A1

Numerator $3x + 12 = 3(x + 4)$, over $(x + 4)$.

3, where $x \neq -4$. The whole denominator cancels, and the restriction is the only trace of it left.

A2

Parentheses first: $(5x - 1) - (3x + 7) = 5x - 1 - 3x - 7 = 2x - 8 = 2(x - 4)$.

$\frac{\mathbf{2(x - 4)}}{\mathbf{x - 2}}$, where $x \neq 2$.

A3

Numerator $x^2 - 25 = (x - 5)(x + 5)$, and $(x - 5)$ cancels.

$\mathbf{x + 5}$, where $x \neq 5$.

A4

Numerator $6x - 6 = 6(x - 1)$ over $(x - 1)(x + 1)$.

$\frac{\mathbf{6}}{\mathbf{x + 1}}$, where $x \neq 1$, $x \neq -1$. Both restrictions stay after the cancel.

A5

Denominator $2x + 6 = 2(x + 3)$. Numerator $(x + 7) - (2x + 1) = -x + 6$.

$\frac{\mathbf{6 - x}}{\mathbf{2(x + 3)}}$, where $x \neq -3$.

Part B

B1

LCD $x(x + 1)$. Numerator $3(x + 1) + 5x = 8x + 3$.

$\frac{\mathbf{8x + 3}}{\mathbf{x(x + 1)}}$, where $x \neq 0$, $x \neq -1$.

B2

LCD $(x - 3)(x + 1)$. Numerator $2(x + 1) + 4(x - 3) = 6x - 10 = 2(3x - 5)$.

$\frac{\mathbf{2(3x - 5)}}{\mathbf{(x - 3)(x + 1)}}$, where $x \neq 3$, $x \neq -1$.

B3

LCD $(x + 2)(x - 4)$. Numerator $5(x - 4) - (3x + 6) = 5x - 20 - 3x - 6 = 2x - 26 = 2(x - 13)$.

$\frac{\mathbf{2(x - 13)}}{\mathbf{(x + 2)(x - 4)}}$, where $x \neq -2$, $x \neq 4$.

B4

LCD $(x + 5)(x - 1)$. Numerator $x(x - 1) + 3(x + 5) = x^2 + 2x + 15$, which does not factor.

$\frac{\mathbf{x^2 + 2x + 15}}{\mathbf{(x + 5)(x - 1)}}$, where $x \neq -5$, $x \neq 1$.

B5

LCD $2x(x + 4)$. The second fraction is built up by $2x$: $7(x + 4) - 6x = x + 28$.

$\frac{\mathbf{x + 28}}{\mathbf{2x(x + 4)}}$, where $x \neq 0$, $x \neq -4$.

Answer key, Parts C and D

Part C

C1

Denominators $(x - 3)(x + 3)$ and $(x + 3)$. LCD $(x - 3)(x + 3)$. Numerator $4 + 2(x - 3) = 2x - 2 = 2(x - 1)$.

$\frac{\mathbf{2(x - 1)}}{\mathbf{(x - 3)(x + 3)}}$, where $x \neq 3$, $x \neq -3$.

C2

Denominators $(x + 2)(x + 3)$ and $(x + 2)$. LCD $(x + 2)(x + 3)$. Numerator $3 - (2x + 6) = -2x - 3$.

$\frac{\mathbf{-(2x + 3)}}{\mathbf{(x + 2)(x + 3)}}$, where $x \neq -2$, $x \neq -3$.

C3

Denominators $(x - 2)(x + 2)$ and $(x - 2)^2$. LCD $(x - 2)^2(x + 2)$, the inventory example from page 1. Numerator $5(x - 2) - (3x + 6) = 5x - 10 - 3x - 6 = 2x - 16 = 2(x - 8)$.

$\frac{\mathbf{2(x - 8)}}{\mathbf{(x - 2)^2(x + 2)}}$, where $x \neq 2$, $x \neq -2$.

C4

Denominators $(x - 3)(x + 2)$ and $(x + 2)$. LCD $(x - 3)(x + 2)$. Numerator $2x + (x - 3) = 3x - 3 = 3(x - 1)$.

$\frac{\mathbf{3(x - 1)}}{\mathbf{(x - 3)(x + 2)}}$, where $x \neq 3$, $x \neq -2$.

C5

Denominators $(x - 2)(x - 3)$ and $(x - 2)(x + 2)$, sharing $(x - 2)$. LCD $(x - 2)(x - 3)(x + 2)$. Numerator $(x + 1)(x + 2) - (x - 1)(x - 3) = (x^2 + 3x + 2) - (x^2 - 4x + 3) = 7x - 1$.

$\frac{\mathbf{7x - 1}}{\mathbf{(x - 2)(x - 3)(x + 2)}}$, where $x \neq 2$, $x \neq 3$, $x \neq -2$.

Part D

D1

Since $7 - x = -(x - 7)$, the second fraction is $-7/(x - 7)$. Numerator $x - 7$, which cancels the denominator.

$\mathbf{1}$, where $x \neq 7$.

D2

The second fraction is $-5/(x - 2)$, and subtracting it adds 5. Numerator $3 + 5 = 8$. Two sign changes in one step, so work slowly here.

$\frac{\mathbf{8}}{\mathbf{x - 2}}$, where $x \neq 2$.

D3

Rewrite $3/(3 - x)$ as $-3/(x - 3)$ and build it up by $(x + 3)$. LCD $(x - 3)(x + 3)$. Numerator $x - 3(x + 3) = -2x - 9$.

$\frac{\mathbf{-(2x + 9)}}{\mathbf{(x - 3)(x + 3)}}$, where $x \neq 3$, $x \neq -3$.

D4

LCD $(x - 1)(x + 1)$. Numerator $(x + 5) - (3x + 3) = -2x + 2 = -2(x - 1)$, and $(x - 1)$ cancels.

$\frac{\mathbf{-2}}{\mathbf{x + 1}}$, where $x \neq 1$, $x \neq -1$.

D5

LCD $(x - 4)(x + 4)$. Numerator $2x - (x + 4) = x - 4$, which cancels the $(x - 4)$ below.

$\dfrac{\mathbf{1}}{\mathbf{x + 4}}$, where $x \neq 4$, $x \neq -4$. Nothing in the answer hints at $x \neq 4$, and it is still required.

Challenge set

E1

LCD $(x - 1)(x + 1)$. Numerator $2(x + 1) + 3(x - 1) - 4 = 5x - 5 = 5(x - 1)$, and $(x - 1)$ cancels.

$\dfrac{\mathbf{5}}{\mathbf{x + 1}}$, where $x \neq 1$, $x \neq -1$.

E2

The first denominator is $(x + 1)^2$, so the LCD is $(x + 1)^2$ and only the second fraction is built up. Numerator $x - (x + 1) = -1$.

$\dfrac{\mathbf{-1}}{\mathbf{(x + 1)^2}}$, where $x \neq -1$.

E3

Rewrite $-2/(3 - x)$ as $+2/(x - 3)$. LCD $(x - 3)(x + 3)$. Numerator $5(x + 3) + 2(x + 3) + (x - 3) = 8x + 18 = 2(4x + 9)$.

$\dfrac{\mathbf{2(4x + 9)}}{\mathbf{(x - 3)(x + 3)}}$, where $x \neq 3$, $x \neq -3$.

WHERE STUDENTS LOSE CREDIT ON THIS SET

1. Adding the denominators instead of building a common one.
2. Distributing the minus sign onto the first term of the second numerator only.
3. Changing a denominator without changing its numerator by the same factor.
4. Using the product of the denominators and leaving the answer unreduced.
5. Treating opposite denominators such as $(x - 5)$ and $(5 - x)$ as different.
6. Dropping a restriction because the factor that caused it canceled.

Full explanations, four worked examples and a step by step animation are at burketutoringinfremont.com/adding-subtracting-rational-expressions-worksheet/