Adding and Subtracting Rational Expressions

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THE RULE

$$\frac{A}{C} + \frac{B}{C} = \frac{A + B}{C} \qquad \frac{A}{C} - \frac{B}{C} = \frac{A - B}{C}$$

The denominators must match before anything is combined, and the shared denominator is written once. Never add the denominators.

SIX STEPS, IN THIS ORDER

1. Factor every denominator.
2. Write the restrictions from those denominators.
3. Build the LCD from the factor inventory.
4. Multiply each fraction up to the LCD, top and bottom.
5. Combine the numerators over one LCD, with the second numerator in parentheses.
6. Simplify, factor, cancel, report with the restrictions.

FACTOR INVENTORY, WORKED ON TWO DENOMINATORS

Denominators $x^2 - 4 = (x - 2)(x + 2)$ and $x^2 - 4x + 4 = (x - 2)^2$.

Distinct factor	Highest power in any one denominator	Goes into the LCD as
$(x - 2)$	2	$(x - 2)^2$
$(x + 2)$	1	$(x + 2)$

LCD = $(x - 2)^2(x + 2)$. A shared factor is counted once, at its highest power. The product of the denominators also works as a common denominator, but it leaves an extra factor to cancel at the end.

SUBTRACTION

Put parentheses around the second numerator, then distribute the minus sign to every term.

$$A - (2x + 5) = A - 2x - 5$$

Every term inside the parentheses changes sign, not only the first.

OPPOSITE DENOMINATORS

$$(a - b) = -(b - a)$$

For a denominator of the form $a - x$, rewrite it as $-(x - a)$. Then $(x - 5)$ and $(5 - x)$ share one LCD factor, and the sign in front of that fraction flips.

WHERE RESTRICTIONS COME FROM

Source	Restriction	Note
Every factor of every original denominator	Set the factor equal to zero and solve	Write these in step 2, before rewriting anything
A factor that cancels later	Still restricted	The original expression was never defined there
The LCD you built	Nothing new	It is made only of factors already listed

Report the restrictions with the answer, not only the simplified expression.

WHAT YOU MAY NOT DO

- Add the denominators. $1/x + 1/y$ is not $2/(x + y)$.
- Change a denominator without changing its numerator by the same factor.
- Cancel a piece that is added rather than multiplied. Only whole factors cancel.
- Drop a restriction because the factor that caused it canceled.

SELF CHECK

Pick a value of x that is not on the restriction list, usually $x = 0$. Evaluate the original expression and your answer separately. The two numbers must match. If they do not, the error sits between those two lines.

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