

Finding the Roots of Quadratics by Factoring

There are multiple ways to find quadratic roots. One of them is to factor. This will not always work but when it does it is usually the quickest way to find the roots. There are different ways to factor and I'll share below how I show my students to factor their quadratics.

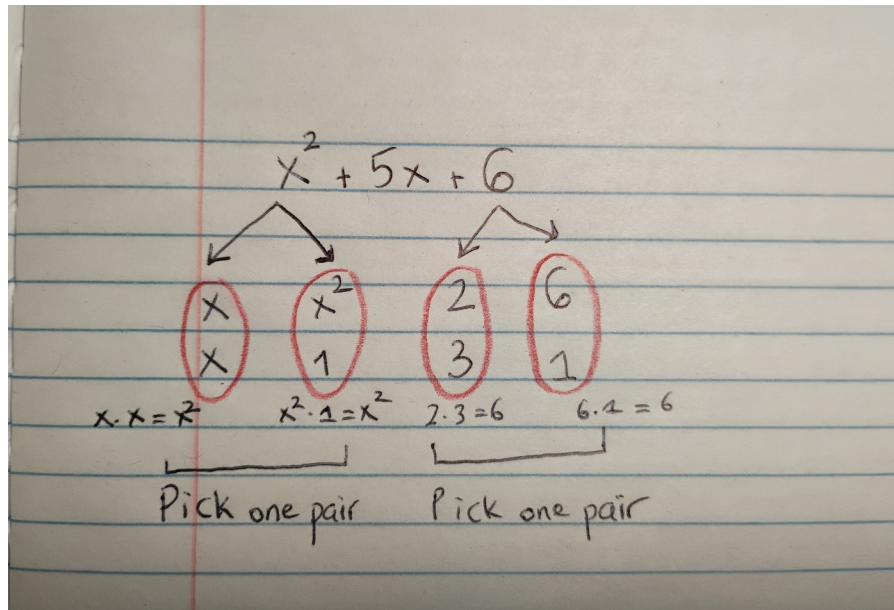
$$Ax^2 + Bx + C$$

For the first term with x^2 we need to come up with the possible product or multiplication pairs to get $1x^2$ below.

$$x \cdot x \text{ or } x^2 \cdot 1 = x^2$$

$$2 \cdot 3 \text{ or } 1 \cdot 6 = 6$$

We do not know which pairs are the correct so we have to some trial and error here and see which one works. We will come across a couple that fail and that's part of the work too!



I put these pairs in a vertical line below the part of the quadratic I'm working with and usually put them in an oval to keep them separate. I pick one pair from the left for x^2 and one pair from the right for the constant 6. Then I draw diagonal arrows between the pairs and multiply them. Do that and sum them up and see if they give you the middle value $5x$ above for the **Bx value**. If not, then you continue to check the other possibilities.

$$\begin{array}{l} x \\ x \end{array} \begin{array}{l} \nearrow 6 \\ \nwarrow 1 \end{array} \quad x \cdot 1 + x \cdot 6 = x + 6x = 7x$$

$$\begin{array}{l} x^2 \\ 1 \end{array} \begin{array}{l} \nearrow 6 \\ \nwarrow 1 \end{array} \quad 1 \cdot x^2 + 6 \cdot 1 = x^2 + 6$$

$$\begin{array}{l} x \\ x \end{array} \begin{array}{l} \nearrow 2 \\ \nwarrow 3 \end{array} \quad x \cdot 3 + x \cdot 2 = 3x + 2x = \boxed{5x}$$

The first two pairs when multiplied with their diagonals gave $7x$. That is not equal to the Bx value of $5x$ in the original quadratic. The second one is also not $5x$ so that is wrong too. The bottom pairs when multiplied with their diagonals give us the $5x$ we were looking for. So, these pairs are the right ones. We are not done yet though.

$$\begin{array}{l} x + 2 \\ x + 3 \end{array} \quad (x+2) \cdot (x+3)$$

Rewrite the vertical pair and then connect the horizontal parts with an oval. This will give you the expressions to put in the parenthesis for your factored quadratic. Now what makes each parenthesis zero?

$$x+2=0 \text{ then } x = -2$$

$$x+3=0 \text{ then } x = -3$$

Practice Problems

Find all real roots by factoring. Show your factored form, then give both roots.

1. $x^2 - 7x + 12 = 0$

9. $x^2 + 12x + 36 = 0$

x = _____

2. $x^2 + 8x + 15 = 0$

x = _____

3. $x^2 - 2x - 8 = 0$

x = _____

4. $x^2 + 5x - 14 = 0$

x = _____

5. $x^2 - 11x + 30 = 0$

x = _____

6. $x^2 - 49 = 0$

x = _____

7. $x^2 - 10x + 25 = 0$

x = _____

8. $x^2 - 36 = 0$

x = _____

x = _____

10. $x^2 - 3x = 0$

x = _____

11. $2x^2 + 5x - 3 = 0$

x = _____

12. $3x^2 - 10x + 8 = 0$

x = _____

13. $2x^2 - 7x + 3 = 0$

x = _____

14. $6x^2 + x - 2 = 0$

x = _____

15. $4x^2 - 12x + 9 = 0$

x = _____

Worked Answer Key

Reasoning shown, not just answers — so you can find where a step went wrong.

1. $x^2 - 7x + 12 = 0$ [Factor]

Product 12, sum $-7 \rightarrow -3$ and -4 . Pairs: $(x - 3)(x - 4) = 0$

Answer: $x = 3, x = 4$

2. $x^2 + 8x + 15 = 0$ [Factor]

Product 15, sum $+8 \rightarrow +3$ and $+5$. Pairs: $(x + 3)(x + 5) = 0$

Answer: $x = -3, x = -5$

3. $x^2 - 2x - 8 = 0$ [Factor]

Product -8 , sum $-2 \rightarrow -4$ and $+2$. Pairs: $(x - 4)(x + 2) = 0$

Answer: $x = 4, x = -2$

4. $x^2 + 5x - 14 = 0$ [Factor]

Product -14 , sum $+5 \rightarrow +7$ and -2 . Pairs: $(x + 7)(x - 2) = 0$

Answer: $x = -7, x = 2$

5. $x^2 - 11x + 30 = 0$ [Factor]

Product 30, sum $-11 \rightarrow -5$ and -6 . Pairs: $(x - 5)(x - 6) = 0$

Answer: $x = 5, x = 6$

6. $x^2 - 49 = 0$ [Difference of squares]

$x^2 - 49 = x^2 - 7^2$. Pairs: $(x - 7)(x + 7) = 0$

Answer: $x = 7, x = -7$

7. $x^2 - 10x + 25 = 0$ [Perfect square]

$25 = 5^2$ and $-10 = 2(-5)$. Pairs: $(x - 5)(x - 5) = 0$

Answer: $x = 5$ (repeated root)

8. $x^2 - 36 = 0$ [Difference of squares]

$x^2 - 36 = x^2 - 6^2$. Pairs: $(x - 6)(x + 6) = 0$

Answer: $x = 6, x = -6$

9. $x^2 + 12x + 36 = 0$ [Perfect square]

$36 = 6^2$ and $12 = 2(6)$. Pairs: $(x + 6)(x + 6) = 0$

Answer: $x = -6$ (repeated root)

10. $x^2 - 3x = 0$ [Common factor]

Factor out x first — don't divide it away. $x(x - 3) = 0$

Answer: $x = 0, x = 3$

11. $2x^2 + 5x - 3 = 0$ [Factor, $a > 1$]

x^2 pairs: $2x \cdot x$. Constant pairs: $-1 \cdot 3$. Diagonals give $+5x$. $(2x - 1)(x + 3) = 0$

Answer: $x = 1/2$, $x = -3$

12. $3x^2 - 10x + 8 = 0$ [Factor, $a > 1$]

x^2 pairs: $3x \cdot x$. Constant pairs: $-4 \cdot -2$. Diagonals give $-10x$. $(3x - 4)(x - 2) = 0$

Answer: $x = 4/3$, $x = 2$

13. $2x^2 - 7x + 3 = 0$ [Factor, $a > 1$]

x^2 pairs: $2x \cdot x$. Constant pairs: $-1 \cdot -3$. Diagonals give $-7x$. $(2x - 1)(x - 3) = 0$

Answer: $x = 1/2$, $x = 3$

14. $6x^2 + x - 2 = 0$ [Factor, $a > 1$]

x^2 pairs: $2x \cdot 3x$. Constant pairs: $-1 \cdot 2$. Diagonals give $+1x$. $(2x - 1)(3x + 2) = 0$

Answer: $x = 1/2$, $x = -2/3$

15. $4x^2 - 12x + 9 = 0$ [Perfect square, $a > 1$]

$4x^2 = (2x)^2$, $9 = 3^2$, $-12x = 2(2x)(-3)$. $(2x - 3)(2x - 3) = 0$

Answer: $x = 3/2$ (repeated root)

Are you still having problems with quadratics or would you like to get a tutor to walk you through each step then. Please reach out for a quick phone consultation and see how we can help with a [Fremont math tutor](#).