

Using the Discriminant to Find the Number of Roots

How $b^2 - 4ac$ tells you how many real solutions a quadratic has — before you solve it

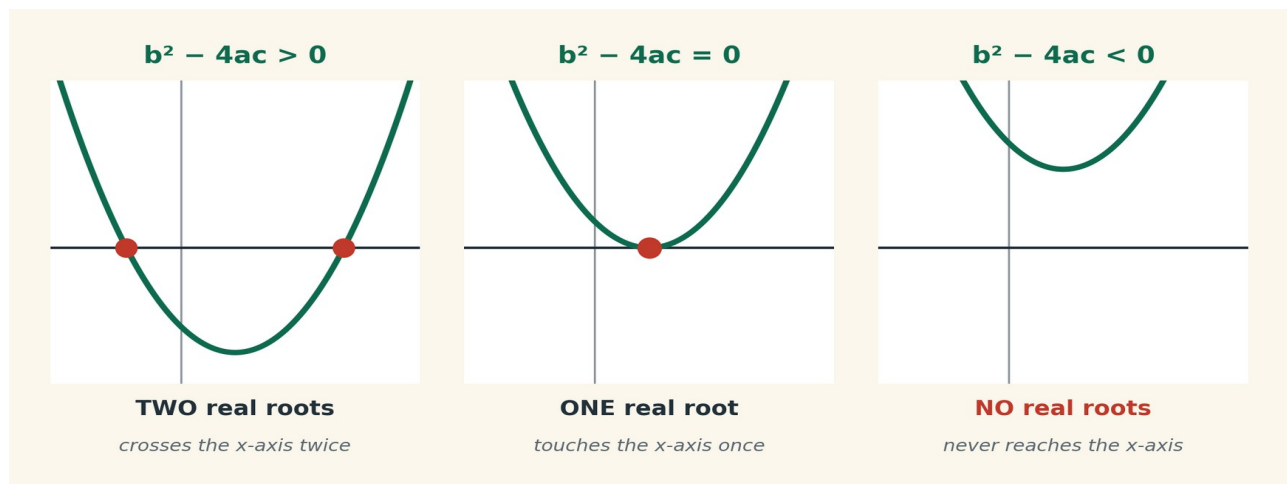
Berke Sahbazoglu · 10+ years tutoring Algebra I & II · Free to print for classroom and home use

Every quadratic in the form $ax^2 + bx + c = 0$ hides one number that answers the question “how many real roots does this have?” That number is the discriminant, and you only need the part of the quadratic formula that lives under the square root.

THE DISCRIMINANT

$$D = b^2 - 4ac$$

$D > 0 \rightarrow$ two real roots $D = 0 \rightarrow$ one real root $D < 0 \rightarrow$ no real roots

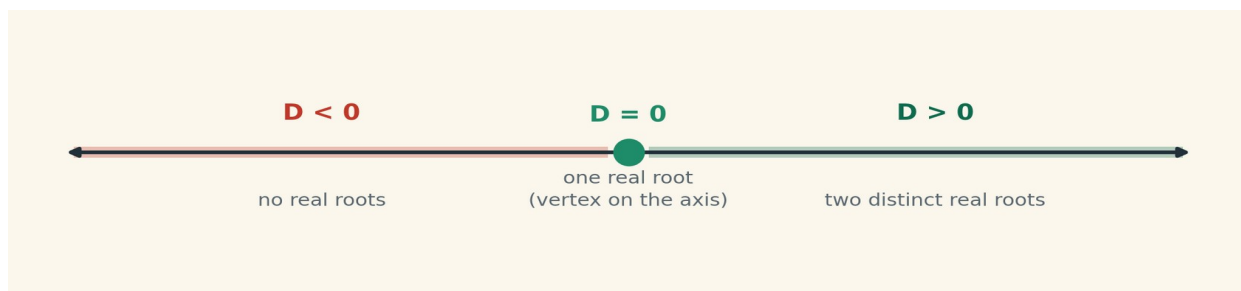


The sign of the discriminant is the same thing as asking how many times the parabola meets the x-axis.

Why This Works

The [quadratic formula](#) is $x = (-b \pm \sqrt{b^2 - 4ac}) / 2a$. Everything that decides the number of roots happens under that square root.

- If $b^2 - 4ac$ is positive, the square root is a real number, and the $\pm$ pushes it out to two different answers.
- If $b^2 - 4ac$ is exactly zero, the square root is 0. Adding and subtracting 0 changes nothing, so the $\pm$ collapses to a single value: $x = -b / 2a$.
- If $b^2 - 4ac$ is negative, you would be taking the square root of a negative number, which no real number does. There are no real roots.



One sign check does all the work. You never have to finish the formula.

The Steps

1. Set the equation equal to zero.

Nothing below is valid until every term is on one side. Move things across first, then read the coefficients.

2. Read off a , b , and c — with their signs.

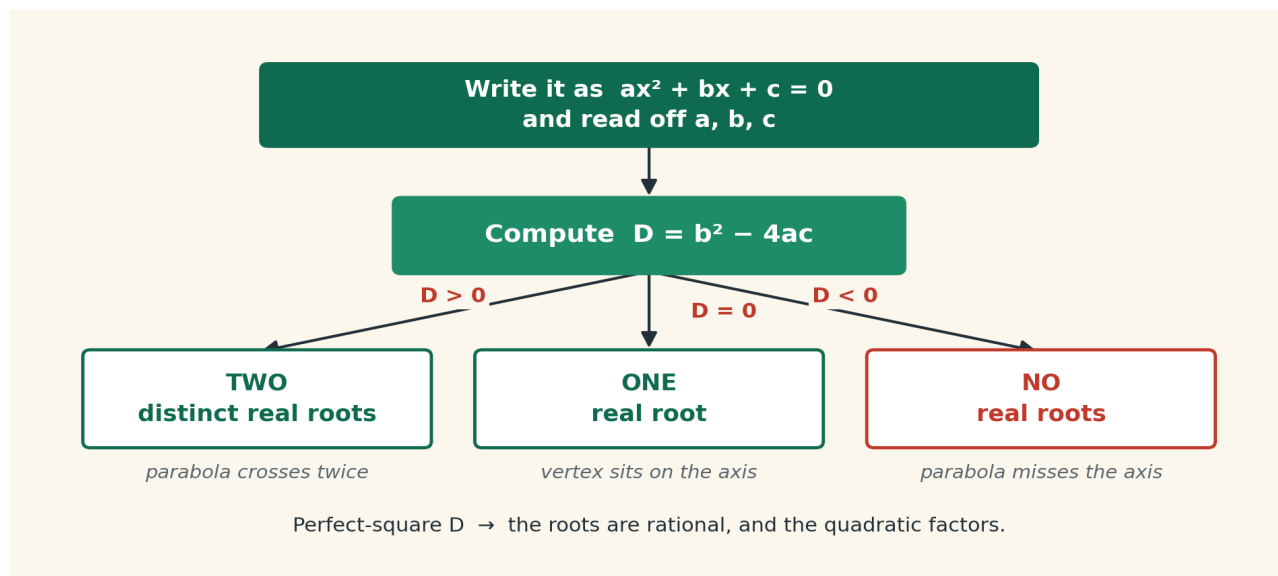
A missing term means that coefficient is zero: in $x^2 - 9 = 0$, $b = 0$. A minus sign in front of a term belongs to the coefficient.

3. Compute $-4ac$ as its own line, before you combine anything.

This is where nearly every discriminant error happens. Writing it separately makes the sign change visible instead of leaving it to happen in your head.

4. Add b^2 and compare to zero.

Positive, zero, or negative — that answers the question. If you also need the roots themselves, only now do you go on to the **full quadratic formula**.



The whole decision, start to finish.

Three Worked Examples

One of each case, worked the way I do it with students.

Example 1 — a positive discriminant

$$2x^2 + 4x - 3 = 0$$

$$a = 2, b = 4, c = -3.$$

$$-4ac = -4 \cdot 2 \cdot (-3) = +24. \text{ Two negatives multiplied give a positive — that is the step to slow down on.}$$

$$D = b^2 - 4ac = 16 + 24 = 40.$$

$40 > 0$, so there are two distinct real roots. 40 is not a perfect square, so those roots will be irrational and will keep a radical in them.

Example 2 — a zero discriminant

$$9x^2 - 12x + 4 = 0$$

$$a = 9, b = -12, c = 4.$$

$$-4ac = -4 \cdot 9 \cdot 4 = -144.$$

$$D = (-12)^2 - 144 = 144 - 144 = 0.$$

$D = 0$, so there is exactly one real root. The $\pm$ contributes nothing, and $x = -b/2a = 12/18 = 2/3$. Geometrically, the vertex is sitting exactly on the x-axis.

Example 3 — a negative discriminant

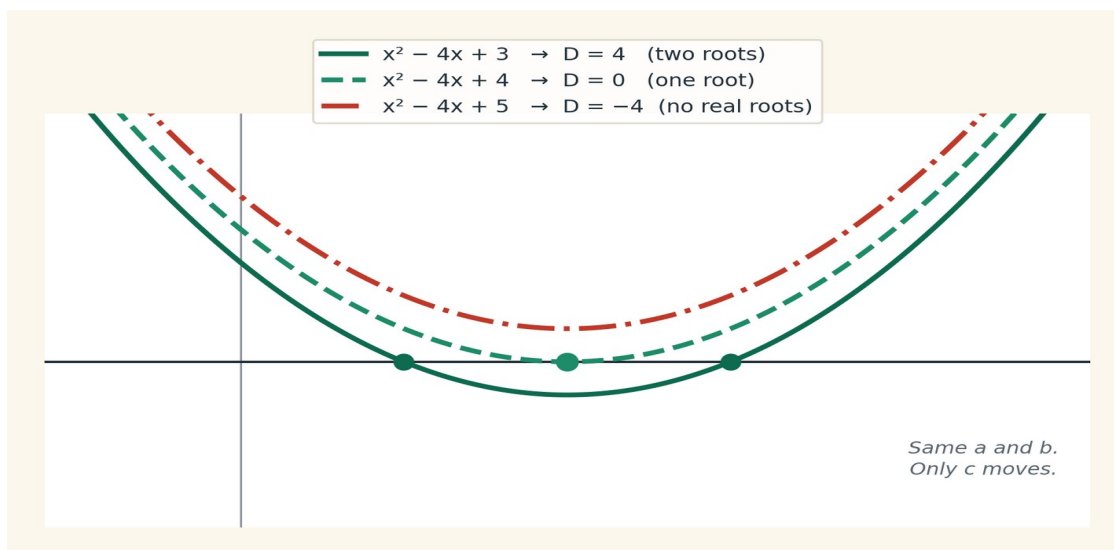
$$x^2 + 2x + 5 = 0$$

$$a = 1, b = 2, c = 5.$$

$$-4ac = -4 \cdot 1 \cdot 5 = -20.$$

$$D = 4 - 20 = -16.$$

$D < 0$, so there are no real roots. Stop here — do not write $\sqrt{-16}$ and keep going. The parabola opens upward with its vertex above the axis, so it never crosses.



Same a and b , three different values of c . Sliding the parabola up by one unit at a time walks it through all three cases.

A Few Things Students Get Wrong Here

Forgetting to set the equation to zero first. If you read coefficients off $x^2 + 3x = 10$, you will get $c = 0$ instead of $c = -10$ and the discriminant will be wrong from the start.

Dropping the minus sign on $-4ac$ when c is negative. When c is negative, $-4ac$ is positive and the discriminant gets larger, not smaller. Write that product on its own line.

Squaring b incorrectly when b is negative. $(-6)^2$ is $+36$, not -36 . The parentheses matter.

Reading “no real roots” as “no solutions.” A negative discriminant means the parabola never crosses the x -axis. There are still two complex solutions, which you will meet in Algebra II.

Assuming a positive discriminant means nice answers. Positive only promises two real roots. Whether they are rational depends on whether D is a perfect square.

BONUS: WHAT THE DISCRIMINANT ALSO TELLS YOU

If D is a perfect square (0, 1, 4, 9, 16, 25, ...), the roots are rational and the quadratic factors over the integers.

If D is positive but not a perfect square, the roots are irrational and factoring will not work — the full quadratic formula is the only route.

Practice Problems

For each quadratic, compute the discriminant and state the number of real roots. You do not need to find the roots themselves.

1. $x^2 + 6x + 5 = 0$

D = _____

Number of real roots: _____

3. $x^2 + x + 1 = 0$

D = _____

Number of real roots: _____

5. $3x^2 + 6x + 3 = 0$

D = _____

Number of real roots: _____

7. $5x^2 + 2x - 1 = 0$

D = _____

Number of real roots: _____

9. $4x^2 - 3x + 2 = 0$

D = _____

Number of real roots: _____

11. $2x^2 + 5x + 4 = 0$

D = _____

Number of real roots: _____

13. $3x^2 - 7x + 1 = 0$

D = _____

Number of real roots: _____

15. $6x^2 - x - 2 = 0$

D = _____

Number of real roots: _____

17. $x^2 + 3x - 1 = 0$

D = _____

Number of real roots: _____

19. $9x^2 - 6x + 1 = 0$

D = _____

Number of real roots: _____

2. $x^2 - 4x + 4 = 0$

D = _____

Number of real roots: _____

4. $2x^2 - 3x - 5 = 0$

D = _____

Number of real roots: _____

6. $x^2 - 2x + 7 = 0$

D = _____

Number of real roots: _____

8. $x^2 - 10x + 25 = 0$

D = _____

Number of real roots: _____

10. $x^2 - 9 = 0$

D = _____

Number of real roots: _____

12. $x^2 + 8x + 16 = 0$

D = _____

Number of real roots: _____

14. $x^2 + 4 = 0$

D = _____

Number of real roots: _____

16. $4x^2 + 12x + 9 = 0$

D = _____

Number of real roots: _____

18. $2x^2 - x + 3 = 0$

D = _____

Number of real roots: _____

20. $x^2 - 5x + 7 = 0$

D = _____

Number of real roots: _____

Going Further

These run the logic backwards: you are given the number of roots and asked for the coefficient.

21. For what values of k does $x^2 + kx + 9 = 0$ have exactly one real root? $k =$ _____
22. For what values of k does $2x^2 + 8x + k = 0$ have two distinct real roots? k _____
23. For what values of k does $x^2 - 6x + k = 0$ have no real roots? k _____
24. For what value of k does $kx^2 + 4x + 1 = 0$ have exactly one real root? (Assume $k \neq 0$.) $k =$ _____

Answer Key

Each entry shows the coefficients, the discriminant computed in full, and the conclusion — so a student can find exactly where their work diverged rather than just checking the final word.

1. $x^2 + 6x + 5 = 0$

$$a = 1, b = 6, c = 5$$

$$D = (6)^2 - 4(1)(5) = 36 - 20 = 16$$

$$16 > 0, \text{ and } 16 \text{ is a perfect square}$$

Answer: Two distinct real roots (both rational)

2. $x^2 - 4x + 4 = 0$

$$a = 1, b = -4, c = 4$$

$$D = (-4)^2 - 4(1)(4) = 16 - 16 = 0$$

$$D = 0$$

Answer: Exactly one real root

3. $x^2 + x + 1 = 0$

$$a = 1, b = 1, c = 1$$

$$D = (1)^2 - 4(1)(1) = 1 - 4 = -3$$

$$D < 0$$

Answer: No real roots

4. $2x^2 - 3x - 5 = 0$

$$a = 2, b = -3, c = -5$$

$$D = (-3)^2 - 4(2)(-5) = 9 + 40 = 49$$

$$49 > 0, \text{ perfect square}$$

Answer: Two distinct real roots (both rational)

5. $3x^2 + 6x + 3 = 0$

$$a = 3, b = 6, c = 3$$

$$D = (6)^2 - 4(3)(3) = 36 - 36 = 0$$

$$D = 0$$

Answer: Exactly one real root

6. $x^2 - 2x + 7 = 0$

$$a = 1, b = -2, c = 7$$

$$D = (-2)^2 - 4(1)(7) = 4 - 28 = -24$$

$$D < 0$$

Answer: No real roots

7. $5x^2 + 2x - 1 = 0$

$$a = 5, b = 2, c = -1$$

$$D = (2)^2 - 4(5)(-1) = 4 + 20 = 24$$

$$24 > 0, \text{ not a perfect square}$$

Answer: Two distinct real roots (irrational)

8. $x^2 - 10x + 25 = 0$

$$a = 1, b = -10, c = 25$$

$$D = (-10)^2 - 4(1)(25) = 100 - 100 = 0$$

$$D = 0$$

Answer: Exactly one real root

9. $4x^2 - 3x + 2 = 0$

$$a = 4, b = -3, c = 2$$

$$D = (-3)^2 - 4(4)(2) = 9 - 32 = -23$$

$$D < 0$$

Answer: No real roots

10. $x^2 - 9 = 0$

$$a = 1, b = 0, c = -9$$

$$D = (0)^2 - 4(1)(-9) = 0 + 36 = 36$$

$$36 > 0, \text{ perfect square}$$

Answer: Two distinct real roots (both rational)

11. $2x^2 + 5x + 4 = 0$

$$a = 2, b = 5, c = 4$$

$$D = (5)^2 - 4(2)(4) = 25 - 32 = -7$$

$$D < 0$$

Answer: No real roots

12. $x^2 + 8x + 16 = 0$

$$a = 1, b = 8, c = 16$$

$$D = (8)^2 - 4(1)(16) = 64 - 64 = 0$$

$$D = 0$$

Answer: Exactly one real root

13. $3x^2 - 7x + 1 = 0$

$$a = 3, b = -7, c = 1$$

$$D = (-7)^2 - 4(3)(1) = 49 - 12 = 37$$

$$37 > 0, \text{ not a perfect square}$$

Answer: Two distinct real roots (irrational)

14. $x^2 + 4 = 0$

$$a = 1, b = 0, c = 4$$

$$D = (0)^2 - 4(1)(4) = 0 - 16 = -16$$

$$D < 0$$

Answer: No real roots

15. $6x^2 - x - 2 = 0$

$$a = 6, b = -1, c = -2$$

$$D = (-1)^2 - 4(6)(-2) = 1 + 48 = 49$$

$$49 > 0, \text{ perfect square}$$

Answer: Two distinct real roots (both rational)

16. $4x^2 + 12x + 9 = 0$

$$a = 4, b = 12, c = 9$$

$$D = (12)^2 - 4(4)(9) = 144 - 144 = 0$$

$$D = 0$$

Answer: Exactly one real root

17. $x^2 + 3x - 1 = 0$

$$a = 1, b = 3, c = -1$$

$$D = (3)^2 - 4(1)(-1) = 9 + 4 = 13$$

$$13 > 0, \text{ not a perfect square}$$

Answer: Two distinct real roots (irrational)

18. $2x^2 - x + 3 = 0$

$$a = 2, b = -1, c = 3$$

$$D = (-1)^2 - 4(2)(3) = 1 - 24 = -23$$

$$D < 0$$

Answer: No real roots

19. $9x^2 - 6x + 1 = 0$

$a = 9, b = -6, c = 1$

$D = (-6)^2 - 4(9)(1) = 36 - 36 = 0$

$D = 0$

Answer: Exactly one real root

20. $x^2 - 5x + 7 = 0$

$a = 1, b = -5, c = 7$

$D = (-5)^2 - 4(1)(7) = 25 - 28 = -3$

$D < 0$

Answer: No real roots

Going Further — Answers

21. $x^2 + kx + 9 = 0$

$D = k^2 - 4(1)(9) = k^2 - 36$. Set $k^2 - 36 = 0$.

Answer: $k = 6$ or $k = -6$

22. $2x^2 + 8x + k = 0$

$D = 64 - 8k$. Two distinct real roots requires $64 - 8k > 0$, so $8k < 64$.

Answer: $k < 8$

23. $x^2 - 6x + k = 0$

$D = 36 - 4k$. No real roots requires $36 - 4k < 0$, so $4k > 36$.

Answer: $k > 9$

24. $kx^2 + 4x + 1 = 0$

$D = 16 - 4k$. Exactly one real root requires $16 - 4k = 0$.

Answer: $k = 4$

TUTOR'S NOTE

When a student gets the discriminant wrong, it is almost never the formula — it is the sign on $-4ac$ when c is negative. Have them write $-4 \cdot a \cdot c$ on its own line every single time, even when it feels unnecessary. The habit costs three seconds and eliminates most of the errors on this page.

RELATED WORKSHEETS

Need the roots themselves, not just the count? [Solving Quadratics with the Quadratic Formula](#)

Discriminant a perfect square? Try factoring first — it's usually faster: [Worksheet: Solving Quadratics by Factoring](#)

Need help with Algebra I or II? I tutor students in Fremont, Newark, and Union City. burketutoringinfremont.com