TEACHER GUIDE

Instructions and Scoring Notes

This activity follows the graph card sort, but it asks for a different kind of thinking. Students are not choosing from finished answers. Each page leaves a different set of boxes open, so they have to generate the missing forms and prove that the pieces agree.

The rule that keeps it honest: a function is not finished until the student checks one point that was not supplied. That catches a wrong leading coefficient even when the zeros and end behavior look right.

Suggested use

Plan	Pages	Time	Good fit
Guided practice	A–B	20–25 min	Right after students learn multiplicity.
Standard lesson	C–E	35–45 min	Algebra 2 review or partner work.
Challenge	F–H	40–55 min	Honors, precalculus review, or extension.

Prerequisites

Students should be able to expand factors, factor a polynomial when reasonable, read intercepts, connect repeated factors to multiplicity, and use degree parity with the leading sign to describe the tails.

Scoring (9 points per organizer)

Evidence	Points	What counts
Equation forms	2	Standard and factored forms are algebraically equivalent.
Table and graph	2	Values are correct; intercept behavior and scale agree.
Zeros and multiplicity	2	Signs are solved correctly and repeated factors are counted.
End behavior and words	2	Both tails and the important intercept behavior are described.
Independent check	1	A new point agrees with the equation and graph or table.

Teaching Notes, Differentiation, and Accuracy Review

Where students usually get stuck

- They copy a factor sign into the zero. Have them write $x + 2 = 0$ before recording $x = -2$.
- They treat every intercept the same. Odd multiplicity crosses; even multiplicity touches and turns. A triple root crosses so slowly that it can look like a touch.
- They infer the leading coefficient from the tails but miss its size. Tails give the sign, not the scale. Use the supplied point or table value to find k.
- They graph from the zeros alone. Require the y-intercept and one more point, or several different curves will fit the same intercept pattern.

Why the “given clue” chips matter

> Pages C and G supply the degree as well as a point. Without a stated degree, a student could legitimately attach an irreducible quadratic factor such as $(x^2 + 1)$ — the zeros, the multiplicities, the end behavior, and even the y-intercept would all still match. The degree chip is what makes the answer unique, and it is worth saying that out loud when you assign those pages.

Differentiation

Support	Standard	Extension
Allow a multiplication grid for expanding. Circle repeated factors before graphing. Complete A–B only.	Assign C–E and require the independent check on every page.	Assign F–H, then ask students to remove one supplied clue and explain whether the function is still uniquely determined.

Accuracy review

All eight standard forms were expanded from the listed factored forms and re-checked by computer algebra before these files were built. Every table value was recomputed from the standard coefficients. Zeros, multiplicities, degree, leading coefficient, end behavior, y-intercepts, and plotted graphs were cross-checked against the same function.

Standards connection

HSA-APR.B.3: identify zeros and use them to sketch graphs. HSA-SSE.A.2: use structure to rewrite expressions. HSF-IF.C.7c: graph polynomial functions and identify zeros when suitable factorizations are available. Mathematical Practices 2 and 3 are used when students connect representations and defend a completed organizer.

Function A: Completed Organizer

Two equation forms are filled in. Use them to build every other view.

Standard form
GIVEN TO STUDENT

$f(x) = x^2 + x - 2$

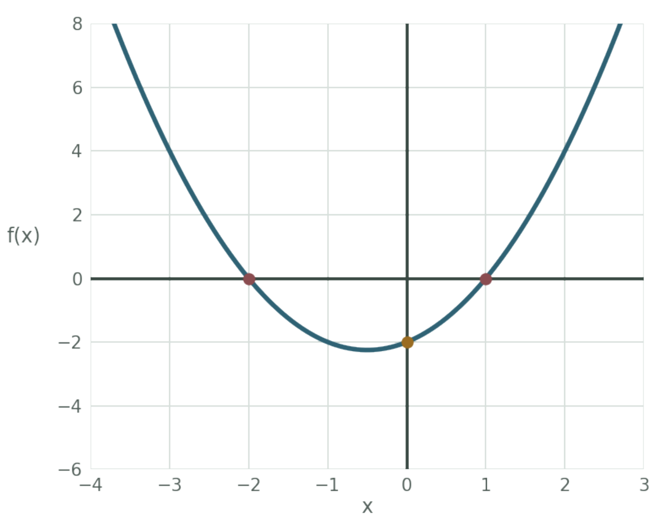
Factored form
GIVEN TO STUDENT

$f(x) = (x + 2)(x - 1)$

Table
STUDENT COMPLETES

x	−3	−2	0	1	2
f(x)	4	0	−2	0	4

Graph
STUDENT COMPLETES



Zeros
STUDENT COMPLETES

$x = -2$ and $x = 1$

Multiplicity and intercept behavior
STUDENT COMPLETES

Both zeros have multiplicity 1. The graph crosses at each one.

End behavior
STUDENT COMPLETES

As $x \to -\infty$, $f(x) \to \infty$. As $x \to \infty$, $f(x) \to \infty$.

Verbal description
STUDENT COMPLETES

An upward-opening quadratic that crosses the x-axis at −2 and 1. It passes through (0, −2).

Fast consistency check. Degree = 2. Leading coefficient = 1. The constant term and the factored form both give $f(0) = -2$.

Why this is the only answer. Both equation forms are handed to you, so every remaining box is forced. Nothing here is a judgement call.

GUIDED • KEY

Function B: Completed Organizer

The graph, table, and degree are complete. Work backward to the equations and structure.

Given clue: Degree 3

Standard form
STUDENT COMPLETES

$g(x) = -x^3 + 3x + 2$

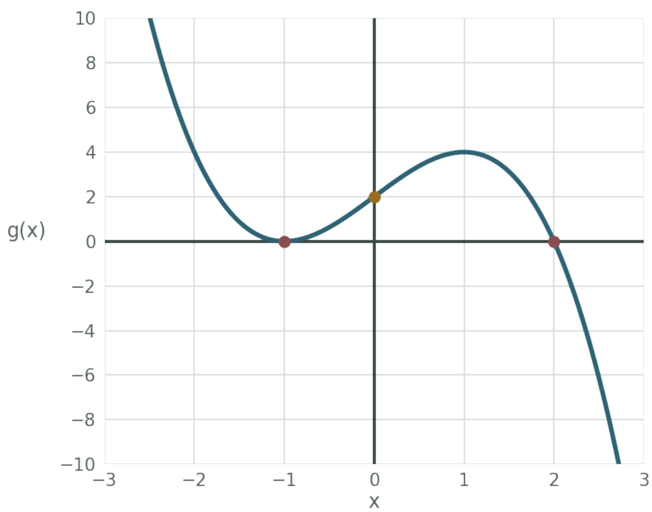
Factored form
STUDENT COMPLETES

$g(x) = -(x + 1)^2(x - 2)$

Table
GIVEN TO STUDENT

x	−2	−1	0	1	2
g(x)	4	0	2	4	0

Graph
GIVEN TO STUDENT



Zeros
STUDENT COMPLETES

$x = -1$ and $x = 2$

Multiplicity and intercept behavior
STUDENT COMPLETES

$x = -1$ has multiplicity 2; $x = 2$ has multiplicity 1.

End behavior
STUDENT COMPLETES

As $x \to -\infty$, $g(x) \to \infty$. As $x \to \infty$, $g(x) \to -\infty$.

Verbal description
STUDENT COMPLETES

A cubic with a negative leading coefficient. It touches at −1, crosses at 2, rises left, and falls right.

Fast consistency check. Degree = 3. Leading coefficient = −1. The constant term and the factored form both give $g(0) = 2$.

Why this is the only answer. A cubic has four unknown coefficients and the table supplies five points, so at most one cubic can fit them all.

STANDARD • KEY

ANSWER KEY — C

Function C: Completed Organizer

Zeros, multiplicities, tails, and one point are enough. Find the scale before expanding.

Given clue: Degree 3

Given clue: $h(0) = 4$

Standard form
STUDENT COMPLETES

$h(x) = 2x^3 - 6x + 4$

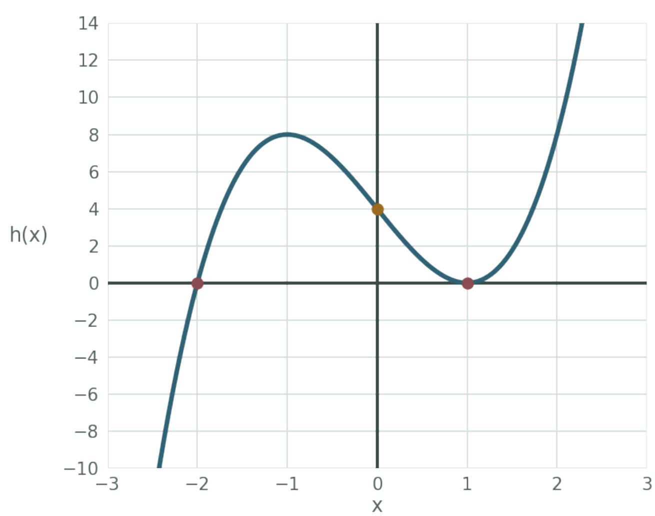
Factored form
STUDENT COMPLETES

$h(x) = 2 \cdot (x + 2)(x - 1)^2$

Table
STUDENT COMPLETES

x	−2	−1	0	1	2
h(x)	0	8	4	0	8

Graph
STUDENT COMPLETES



Zeros
GIVEN TO STUDENT

$x = -2$ and $x = 1$

Multiplicity and intercept behavior
GIVEN TO STUDENT

$x = -2$ has multiplicity 1; $x = 1$ has multiplicity 2.

End behavior
GIVEN TO STUDENT

As $x \to -\infty$, $h(x) \to -\infty$. As $x \to \infty$, $h(x) \to \infty$.

Verbal description
STUDENT COMPLETES

A positive cubic. It crosses at −2, touches and turns at 1, and has y-intercept 4.

Fast consistency check. Degree = 3. Leading coefficient = 2. The constant term and the factored form both give $h(0) = 4$.

Why this is the only answer. The multiplicities add to $1 + 2 = 3$, which matches the given degree, so no extra factor can hide in the function. $h(0) = 4$ gives $4 = k(2)(1)$, so $k = 2$.

STANDARD • KEY

Function D: Completed Organizer

The standard form hides the zeros. The graph gives you a place to start factoring.

Standard form
GIVEN TO STUDENT

$p(x) = -x^4 + 5x^2 - 4$

Factored form
STUDENT COMPLETES

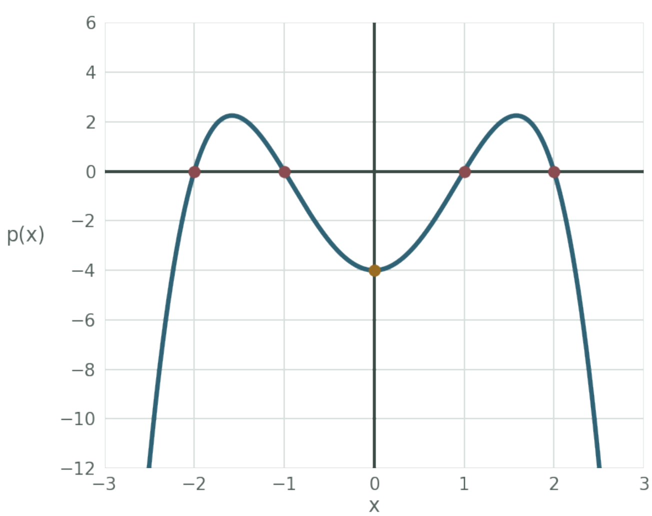
$p(x) = -(x + 2)(x + 1)(x - 1)(x - 2)$

Table
STUDENT COMPLETES

x	−3	−2	−1	0	1	2	3
p(x)	−40	0	0	−4	0	0	−40

(−3, −40) and (3, −40) sit below the printed grid. Record them in the table only.

Graph
GIVEN TO STUDENT



Zeros
STUDENT COMPLETES

$x = -2, -1, 1,$ and 2

Multiplicity and intercept behavior
STUDENT COMPLETES

All four zeros have multiplicity 1. The graph crosses at each one.

End behavior
STUDENT COMPLETES

As $x \to -\infty$, $p(x) \to -\infty$. As $x \to \infty$, $p(x) \to -\infty$.

Verbal description
STUDENT COMPLETES

A negative quartic with four simple zeros at −2, −1, 1, and 2. Both tails fall.

Fast consistency check. Degree = 4. Leading coefficient = −1. The constant term and the factored form both give $p(0) = -4$.

Why this is the only answer. The standard form is supplied, so the factorization is a fact to be found rather than a choice to be made.

STANDARD • KEY

ANSWER KEY — E

Function E: Completed Organizer

The factors show the intercept behavior. Use the table to check the height and symmetry.

Standard form
STUDENT COMPLETES

$q(x) = x^4 - 2x^3 - 3x^2 + 4x + 4$

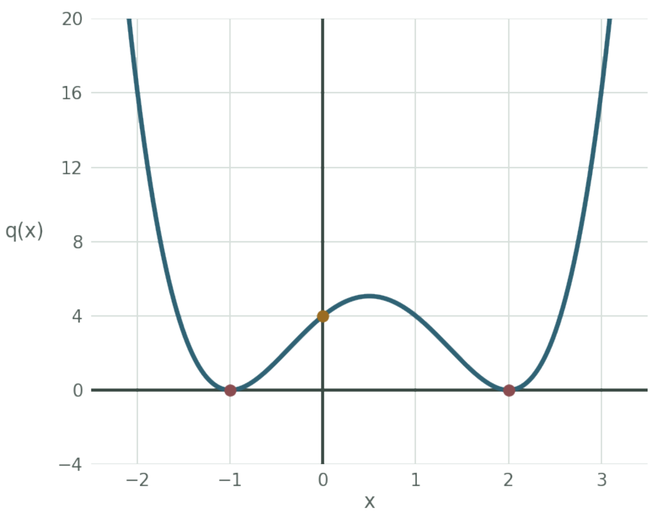
Factored form
GIVEN TO STUDENT

$q(x) = (x + 1)^2(x - 2)^2$

Table
GIVEN TO STUDENT

x	−2	−1	0	1	2	3
q(x)	16	0	4	4	0	16

Graph
STUDENT COMPLETES



Zeros
STUDENT COMPLETES

$x = -1$ and $x = 2$

Multiplicity and intercept behavior
STUDENT COMPLETES

Both zeros have multiplicity 2. The graph touches and turns at each one.

End behavior
STUDENT COMPLETES

As $x \to -\infty$, $q(x) \to \infty$. As $x \to \infty$, $q(x) \to \infty$.

Verbal description
STUDENT COMPLETES

A positive quartic that touches at −1 and 2. Both tails rise, and the y-intercept is 4.

Fast consistency check. Degree = 4. Leading coefficient = 1. The constant term and the factored form both give $q(0) = 4$.

Why this is the only answer. The factored form is supplied, so expanding it is the only route to the standard form.

CHALLENGE • KEY

ANSWER KEY — F

Function F: Completed Organizer

Turn the description into factors first. The y-intercept determines the leading coefficient.

Given clue: $r(0) = -2$

Standard form
STUDENT COMPLETES

$r(x) = (1/2)x^5 + (1/2)x^4 - (5/2)x^3 - (1/2)x^2 + 4x - 2$

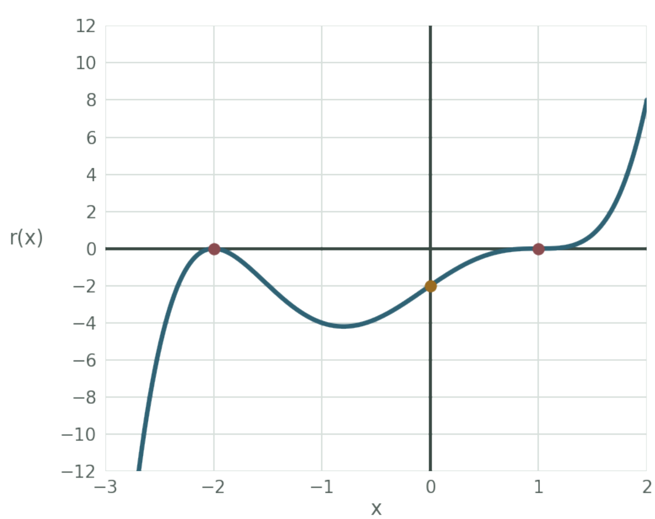
Factored form
STUDENT COMPLETES

$r(x) = (1/2) \cdot (x + 2)^2(x - 1)^3$

Table
STUDENT COMPLETES

x	−2	−1	0	1	2
r(x)	0	−4	−2	0	8

Graph
STUDENT COMPLETES



Zeros
STUDENT COMPLETES

$x = -2$ and $x = 1$

Multiplicity and intercept behavior
STUDENT COMPLETES

$x = -2$ has multiplicity 2; $x = 1$ has multiplicity 3.

End behavior
STUDENT COMPLETES

As $x \to -\infty$, $r(x) \to -\infty$. As $x \to \infty$, $r(x) \to \infty$.

Verbal description
GIVEN TO STUDENT

A positive fifth-degree function. It touches at −2, crosses with flattening at 1, and passes through (0, −2).

Fast consistency check. Degree = 5. Leading coefficient = 1/2. The constant term and the factored form both give $r(0) = -2$.

Why this is the only answer. “Fifth-degree” fixes the total. Flattening at a crossing means an odd multiplicity of at least 3, so $x = 1$ takes 3 and the touch at −2 takes the remaining 2. $r(0) = -2$ gives $-2 = k(4)(-1)$, so $k = 1/2$.

CHALLENGE • KEY

Function G: Completed Organizer

Build $k(x - r)^m$ from the structure, then use the point to solve for k.

Given clue: Degree 5

Given clue: $s(0) = -4$

Standard form
STUDENT COMPLETES

$s(x) = -x^5 + x^4 + 5x^3 - x^2 - 8x - 4$

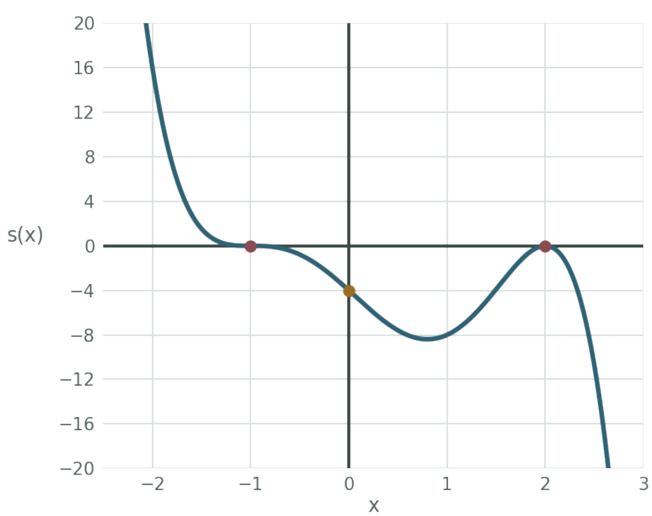
Factored form
STUDENT COMPLETES

$s(x) = -(x + 1)^3(x - 2)^2$

Table
STUDENT COMPLETES

x	−2	−1	0	1	2
s(x)	16	0	−4	−8	0

Graph
STUDENT COMPLETES



Zeros
GIVEN TO STUDENT

$x = -1$ and $x = 2$

Multiplicity and intercept behavior
GIVEN TO STUDENT

$x = -1$ has multiplicity 3; $x = 2$ has multiplicity 2.

End behavior
GIVEN TO STUDENT

As $x \to -\infty$, $s(x) \to \infty$. As $x \to \infty$, $s(x) \to -\infty$.

Verbal description
STUDENT COMPLETES

A negative fifth-degree function. It crosses with flattening at −1, touches at 2, rises left, and falls right.

Fast consistency check. Degree = 5. Leading coefficient = −1. The constant term and the factored form both give $s(0) = -4$.

Why this is the only answer. The multiplicities add to 3 + 2 = 5, which matches the given degree, so no extra factor fits. $s(0) = -4$ gives $-4 = k(1)(4)$, so $k = -1$.

CHALLENGE • KEY

ANSWER KEY — H

Function H: Completed Organizer

Only the factored form is supplied. Complete all seven remaining representations.

Standard form

STUDENT COMPLETES

$t(x) = (1/2)x^4 + (3/2)x^3 - (3/2)x^2 - (11/2)x - 3$

Factored form

GIVEN TO STUDENT

$t(x) = (1/2) \cdot (x + 3)(x + 1)^2(x - 2)$

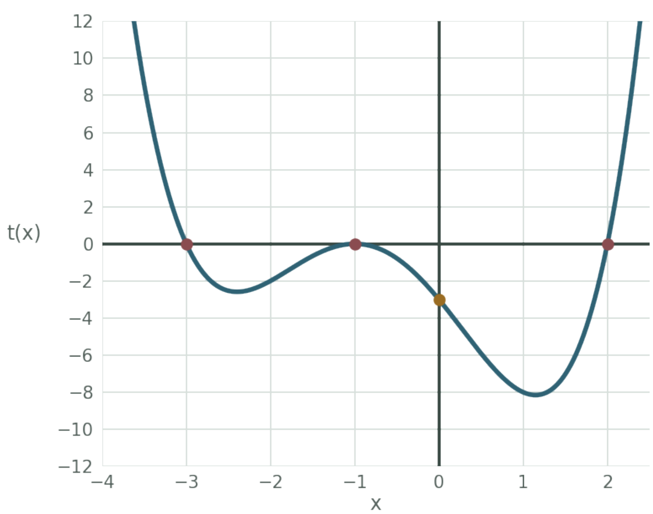
Table

STUDENT COMPLETES

x	−3	−2	−1	0	1	2
t(x)	0	−2	0	−3	−8	0

Graph

STUDENT COMPLETES



Zeros

STUDENT COMPLETES

x = −3, −1, and 2

Multiplicity and intercept behavior

STUDENT COMPLETES

x = −3 and x = 2 have multiplicity 1; x = −1 has multiplicity 2.

End behavior

STUDENT COMPLETES

As x → −∞, t(x) → ∞. As x → ∞, t(x) → ∞.

Verbal description

STUDENT COMPLETES

A positive quartic. It crosses at −3 and 2, touches at −1, passes through (0, −3), and has both tails rising.

Fast consistency check. Degree = 4. Leading coefficient = 1/2. The constant term and the factored form both give t(0) = −3.

Why this is the only answer. The factored form is supplied, so every other box follows from expanding it and reading the factors.