

Polynomial Multiple-Representations Activity

Students do not match finished cards. They complete missing representations, explain what each clue proves, and check that all eight views describe the same function.

8 organizers

quadratic through fifth degree

3 levels

guided, standard, challenge

25-55 min

one lesson or selected pages

Student routine

1. Mark the boxes that were given to you.
2. Complete the easiest missing representation first.
3. Move back and forth between factors, zeros, graph behavior, and end behavior.
4. Use one extra test point before calling the organizer finished.

Read the given clues first. Some pages hand you the degree or a single point. Those are not decoration — without them the function would not be pinned down, and the leading coefficient could be any size.

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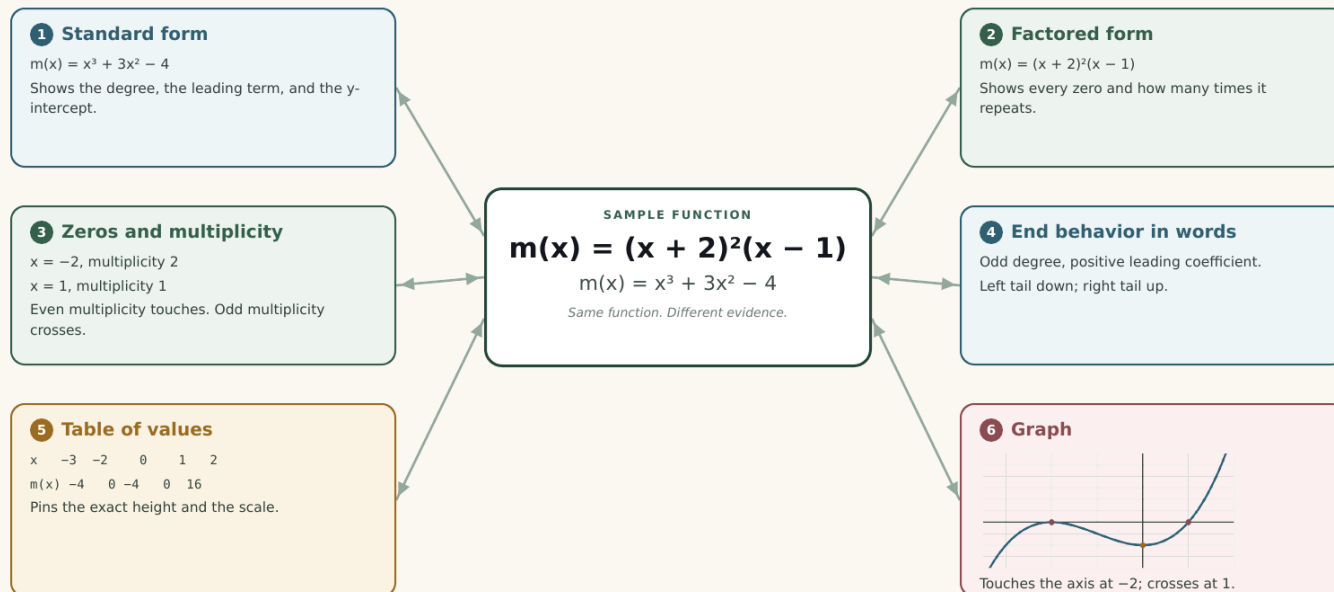
<https://burketutoringinfremont.com/polynomial-multiple-representations-worksheet/>

Reference: Six Ways to Understand a Polynomial Function

Keep this page beside the activity. Every arrow runs both directions, so any view can be used to check any other.

Six Ways to Understand a Polynomial Function

One function. Six connected views. Every arrow runs both directions — each view can check any other.



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What each view proves

- **Standard form** gives the degree, the leading term, and the y-intercept at a glance.
- **Factored form** gives every zero and its multiplicity.
- **Zeros and multiplicity** decide crossing, touching, and flattening.

What no single view proves

- **End behavior** gives the *sign* of the leading coefficient, never its size.
- **A graph** without a labelled point cannot pin the vertical scale.
- That is why one supplied point, or the y-intercept, does the last piece of work on every sparse page.

Function A: Complete the Missing Representations

Two equation forms are filled in. Use them to build every other view.

Standard form

GIVEN

$$f(x) = x^2 + x - 2$$

Factored form

GIVEN

$$f(x) = (x + 2)(x - 1)$$

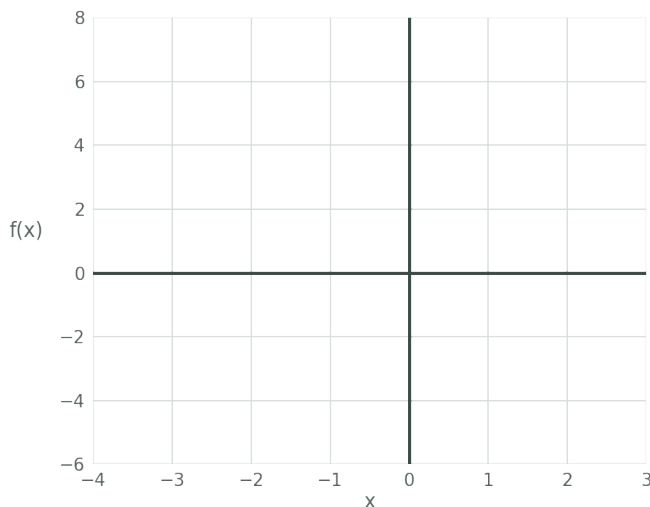
Table

COMPLETE

x	-3	-2	0	1	2
f(x)					

Graph

COMPLETE



Zeros

COMPLETE

Multiplicity and intercept behavior

COMPLETE

End behavior

COMPLETE

Verbal description

COMPLETE

One check before you move on

Choose a point that was not handed to you.
Substitute it into your equation and make sure it
lands on your table or graph.

x = _____ output = _____ agrees? _____

Function B: Complete the Missing Representations

The graph, table, and degree are complete. Work backward to the equations and structure.

Given clue: Degree 3

Standard form

COMPLETE

Zeros

COMPLETE

Factored form

COMPLETE

Multiplicity and intercept behavior

COMPLETE

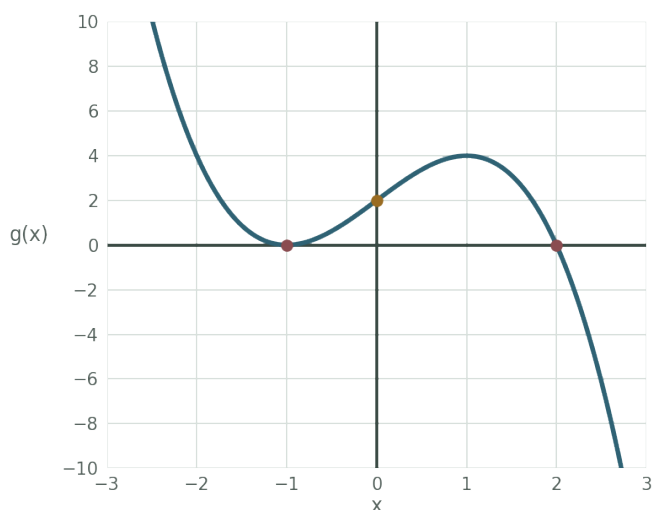
Table

GIVEN

x	-2	-1	0	1	2
g(x)	4	0	2	4	0

Graph

GIVEN



End behavior

COMPLETE

Verbal description

COMPLETE

One check before you move on

Choose a point that was not handed to you.
Substitute it into your equation and make sure it
lands on your table or graph.

x = _____ output = _____ agrees? _____

Function C: Complete the Missing Representations

Zeros, multiplicities, tails, and one point are enough. Find the scale before expanding.

Given clue: Degree 3

Given clue: $h(0) = 4$

Standard form

COMPLETE

Factored form

COMPLETE

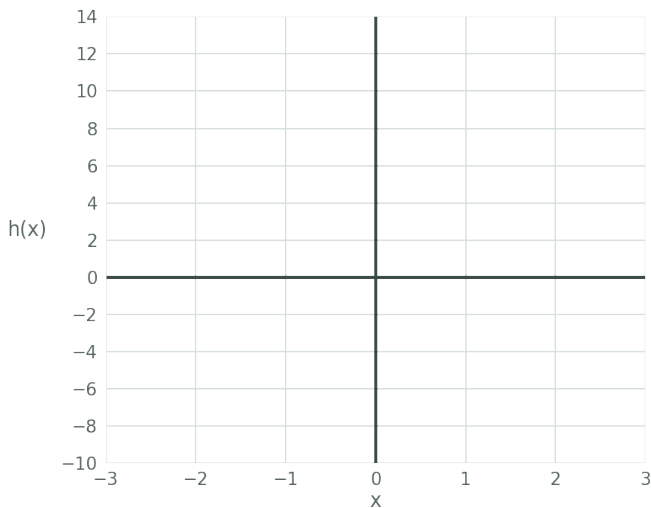
Table

COMPLETE

x	-2	-1	0	1	2
h(x)					

Graph

COMPLETE



Zeros

GIVEN

$x = -2$ and $x = 1$

Multiplicity and intercept behavior

GIVEN

$x = -2$ has multiplicity 1; $x = 1$ has multiplicity 2.

End behavior

GIVEN

As $x \rightarrow -\infty$, $h(x) \rightarrow -\infty$. As $x \rightarrow \infty$, $h(x) \rightarrow \infty$.

Verbal description

COMPLETE

One check before you move on

Choose a point that was not handed to you.

Substitute it into your equation and make sure it lands on your table or graph.

$x = \underline{\hspace{1cm}}$ output = $\underline{\hspace{1cm}}$ agrees? $\underline{\hspace{1cm}}$

Function D: Complete the Missing Representations

The standard form hides the zeros. The graph gives you a place to start factoring.

Standard form

GIVEN

$$p(x) = -x^4 + 5x^2 - 4$$

Zeros

COMPLETE

Multiplicity and intercept behavior

COMPLETE

End behavior

COMPLETE

Verbal description

COMPLETE

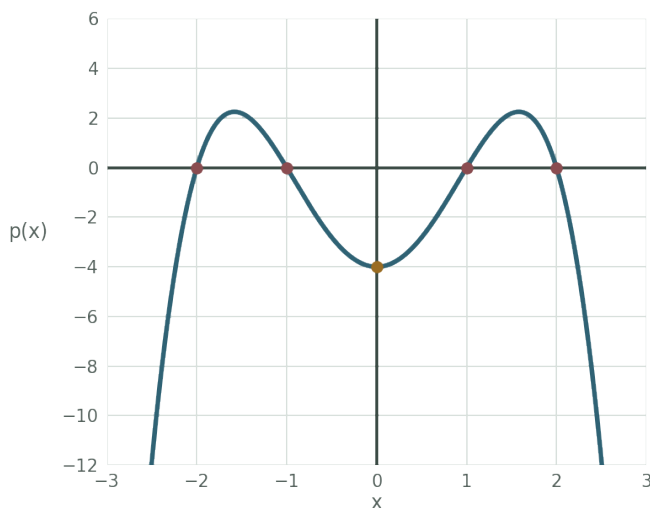
Table

COMPLETE

x	-3	-2	-1	0	1	2	3
p(x)							

Graph

GIVEN



One check before you move on

Choose a point that was not handed to you.

Substitute it into your equation and make sure it lands on your table or graph.

x = _____ output = _____ agrees? _____

Function E: Complete the Missing Representations

The factors show the intercept behavior. Use the table to check the height and symmetry.

Standard form
COMPLETE

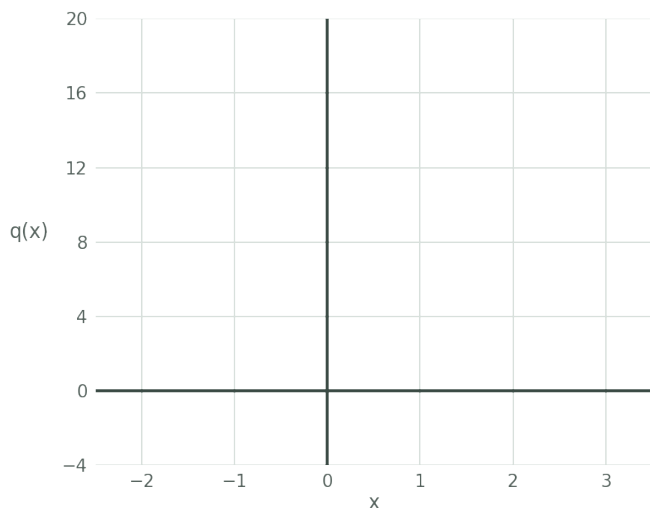
Zeros
COMPLETE

Factored form
GIVEN

$$q(x) = (x + 1)^2(x - 2)^2$$

Table
GIVEN

x	-2	-1	0	1	2	3
q(x)	16	0	4	4	0	16

Graph
COMPLETE

Multiplicity and intercept behavior
COMPLETE

End behavior
COMPLETE

Verbal description
COMPLETE

One check before you move on

Choose a point that was not handed to you.
Substitute it into your equation and make sure it
lands on your table or graph.

x = _____ output = _____ agrees? _____

CHALLENGE

Name _____

Date _____

Function F: Complete the Missing Representations

Turn the description into factors first. The y-intercept determines the leading coefficient.

Given clue: $r(0) = -2$

Standard form

COMPLETE

Zeros

COMPLETE

Factored form

COMPLETE

Multiplicity and intercept behavior

COMPLETE

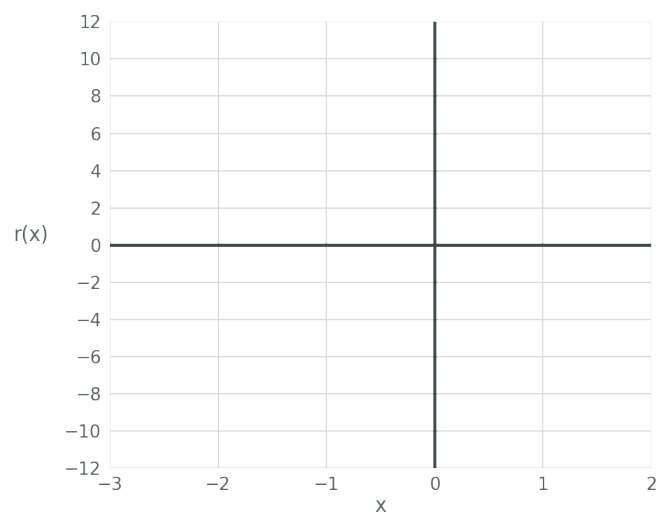
Table

COMPLETE

x	-2	-1	0	1	2
r(x)					

Graph

COMPLETE



End behavior

COMPLETE

Verbal description

GIVEN

A positive fifth-degree function. It touches at -2 , crosses with flattening at 1 , and passes through $(0, -2)$.

One check before you move on

Choose a point that was not handed to you.
Substitute it into your equation and make sure it lands on your table or graph.

$x =$ _____ $\text{output} =$ _____ agrees? _____

CHALLENGE

Name _____

Date _____

Function G: Complete the Missing Representations

Build $k(x - r)^m$ from the structure, then use the point to solve for k .

Given clue: Degree 5

Given clue: $s(0) = -4$

Standard form

COMPLETE

Factored form

COMPLETE

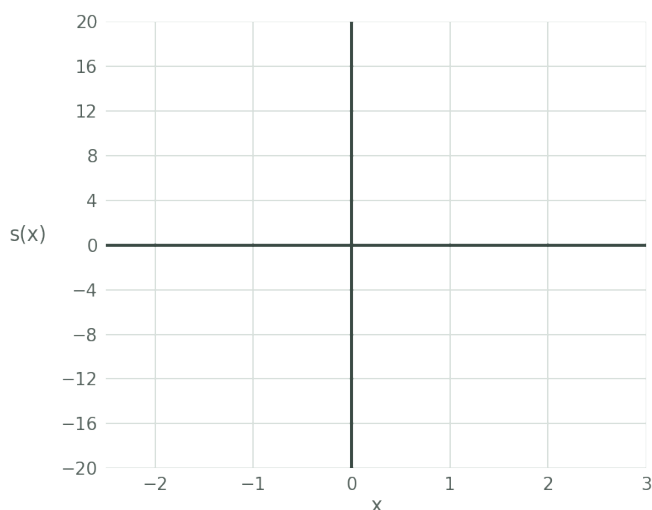
Table

COMPLETE

x	-2	-1	0	1	2
s(x)					

Graph

COMPLETE



Zeros

GIVEN

$x = -1$ and $x = 2$

Multiplicity and intercept behavior

GIVEN

$x = -1$ has multiplicity 3; $x = 2$ has multiplicity 2.

End behavior

GIVEN

As $x \rightarrow -\infty$, $s(x) \rightarrow \infty$. As $x \rightarrow \infty$, $s(x) \rightarrow -\infty$.

Verbal description

COMPLETE

One check before you move on

Choose a point that was not handed to you.
Substitute it into your equation and make sure it
lands on your table or graph.

$x = \underline{\hspace{2cm}}$ output = $\underline{\hspace{2cm}}$ agrees? $\underline{\hspace{2cm}}$

CHALLENGE

Name _____

Date _____

Function H: Complete the Missing Representations

Only the factored form is supplied. Complete all seven remaining representations.

Standard form

COMPLETE

Zeros

COMPLETE

Factored form

GIVEN

$$t(x) = (1/2) \cdot (x + 3)(x + 1)^2(x - 2)$$

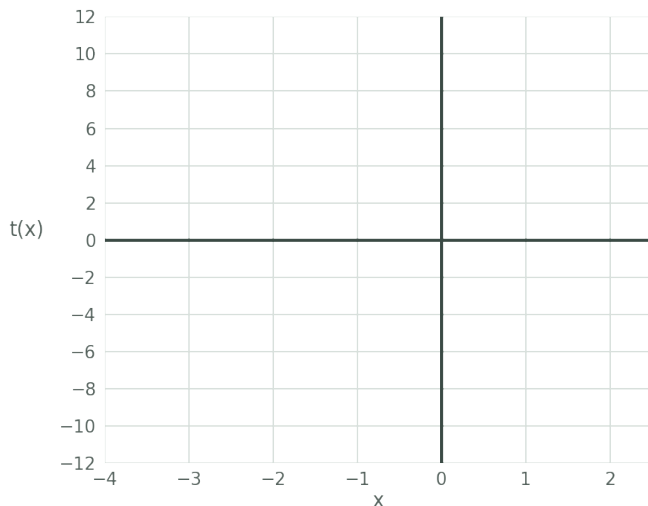
Table

COMPLETE

x	-3	-2	-1	0	1	2
t(x)						

Graph

COMPLETE



Multiplicity and intercept behavior

COMPLETE

End behavior

COMPLETE

Verbal description

COMPLETE

One check before you move on

Choose a point that was not handed to you.
Substitute it into your equation and make sure it
lands on your table or graph.

x = _____ output = _____ agrees? _____

Instructions and Scoring Notes

This activity follows the graph card sort, but it asks for a different kind of thinking. Students are not choosing from finished answers. Each page leaves a different set of boxes open, so they have to generate the missing forms and prove that the pieces agree.

The rule that keeps it honest: a function is not finished until the student checks one point that was not supplied. That catches a wrong leading coefficient even when the zeros and end behavior look right.

Suggested use

Plan	Pages	Time	Good fit
Guided practice	A–B	20–25 min	Right after students learn multiplicity.
Standard lesson	C–E	35–45 min	Algebra 2 review or partner work.
Challenge	F–H	40–55 min	Honors, precalculus review, or extension.

Prerequisites

Students should be able to expand factors, factor a polynomial when reasonable, read intercepts, connect repeated factors to multiplicity, and use degree parity with the leading sign to describe the tails.

Scoring (9 points per organizer)

Evidence	Points	What counts
Equation forms	2	Standard and factored forms are algebraically equivalent.
Table and graph	2	Values are correct; intercept behavior and scale agree.
Zeros and multiplicity	2	Signs are solved correctly and repeated factors are counted.
End behavior and words	2	Both tails and the important intercept behavior are described.
Independent check	1	A new point agrees with the equation and graph or table.

Teaching Notes, Differentiation, and Accuracy Review

Where students usually get stuck

- They copy a factor sign into the zero. Have them write $x + 2 = 0$ before recording $x = -2$.
- They treat every intercept the same. Odd multiplicity crosses; even multiplicity touches and turns. A triple root crosses so slowly that it can look like a touch.
- They infer the leading coefficient from the tails but miss its size. Tails give the sign, not the scale. Use the supplied point or table value to find k .
- They graph from the zeros alone. Require the y -intercept and one more point, or several different curves will fit the same intercept pattern.

Why the “given clue” chips matter

Pages C and G supply the degree as well as a point. Without a stated degree, a student could legitimately attach an irreducible quadratic factor such as $(x^2 + 1)$ — the zeros, the multiplicities, the end behavior, and even the y -intercept would all still match. The degree chip is what makes the answer unique, and it is worth saying that out loud when you assign those pages.

Differentiation

Support	Standard	Extension
Allow a multiplication grid for expanding. Circle repeated factors before graphing. Complete A–B only.	Assign C–E and require the independent check on every page.	Assign F–H, then ask students to remove one supplied clue and explain whether the function is still uniquely determined.

Accuracy review

All eight standard forms were expanded from the listed factored forms and re-checked by computer algebra before these files were built. Every table value was recomputed from the standard coefficients. Zeros, multiplicities, degree, leading coefficient, end behavior, y -intercepts, and plotted graphs were cross-checked against the same function.

Standards connection

HSA-APR.B.3: identify zeros and use them to sketch graphs. HSA-SSE.A.2: use structure to rewrite expressions. HSF-IF.C.7c: graph polynomial functions and identify zeros when suitable factorizations are available. Mathematical Practices 2 and 3 are used when students connect representations and defend a completed organizer.

Function A: Completed Organizer

Two equation forms are filled in. Use them to build every other view.

Standard form

GIVEN TO STUDENT

$$f(x) = x^2 + x - 2$$

Zeros

STUDENT COMPLETES

$$x = -2 \text{ and } x = 1$$

Factored form

GIVEN TO STUDENT

$$f(x) = (x + 2)(x - 1)$$

Multiplicity and intercept behavior

STUDENT COMPLETES

Both zeros have multiplicity 1. The graph crosses at each one.

Table

STUDENT COMPLETES

x	-3	-2	0	1	2
f(x)	4	0	-2	0	4

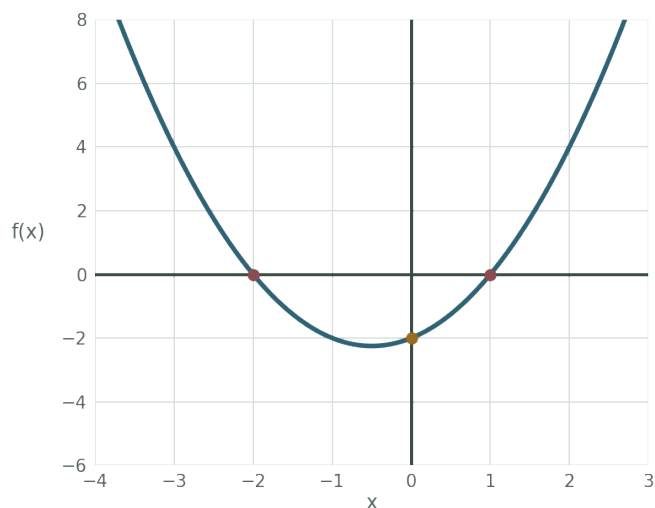
End behavior

STUDENT COMPLETES

As $x \rightarrow -\infty$, $f(x) \rightarrow \infty$. As $x \rightarrow \infty$, $f(x) \rightarrow \infty$.

Graph

STUDENT COMPLETES



Verbal description

STUDENT COMPLETES

An upward-opening quadratic that crosses the x-axis at -2 and 1. It passes through (0, -2).

Fast consistency check. Degree = 2. Leading coefficient = 1. The constant term and the factored form both give $f(0) = -2$.

Why this is the only answer. Both equation forms are handed to you, so every remaining box is forced. Nothing here is a judgement call.

Function B: Completed Organizer

The graph, table, and degree are complete. Work backward to the equations and structure.

Given clue: Degree 3

Standard form

STUDENT COMPLETES

$$g(x) = -x^3 + 3x + 2$$

Factored form

STUDENT COMPLETES

$$g(x) = -(x + 1)^2(x - 2)$$

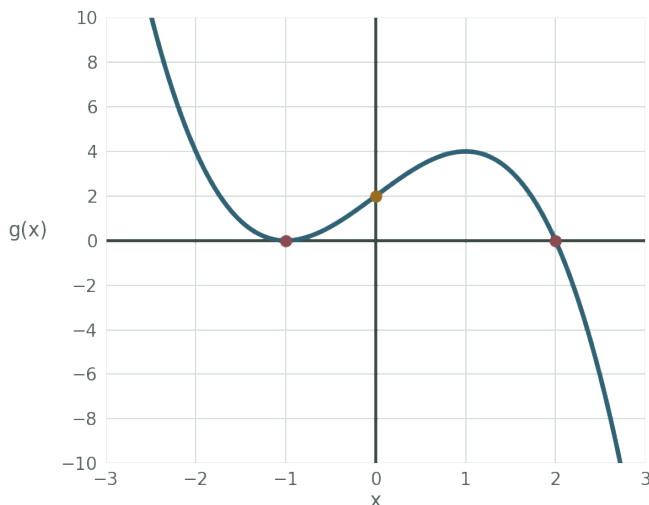
Table

GIVEN TO STUDENT

x	-2	-1	0	1	2
g(x)	4	0	2	4	0

Graph

GIVEN TO STUDENT



Zeros

STUDENT COMPLETES

$$x = -1 \text{ and } x = 2$$

Multiplicity and intercept behavior

STUDENT COMPLETES

$x = -1$ has multiplicity 2; $x = 2$ has multiplicity 1.

End behavior

STUDENT COMPLETES

As $x \rightarrow -\infty$, $g(x) \rightarrow \infty$. As $x \rightarrow \infty$, $g(x) \rightarrow -\infty$.

Verbal description

STUDENT COMPLETES

A cubic with a negative leading coefficient. It touches at -1 , crosses at 2 , rises left, and falls right.

Fast consistency check. Degree = 3. Leading coefficient = -1 . The constant term and the factored form both give $g(0) = 2$.

Why this is the only answer. A cubic has four unknown coefficients and the table supplies five points, so at most one cubic can fit them all.

Function C: Completed Organizer

Zeros, multiplicities, tails, and one point are enough. Find the scale before expanding.

Given clue: Degree 3

Given clue: $h(0) = 4$

Standard form

STUDENT COMPLETES

$$h(x) = 2x^3 - 6x + 4$$

Factored form

STUDENT COMPLETES

$$h(x) = 2 \cdot (x + 2)(x - 1)^2$$

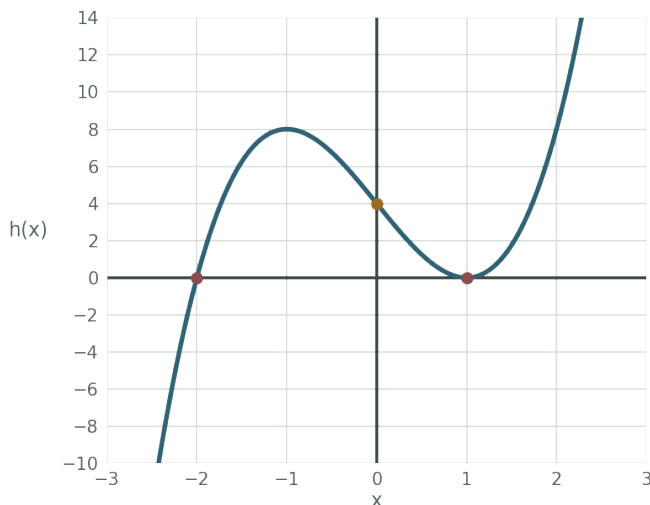
Table

STUDENT COMPLETES

x	-2	-1	0	1	2
h(x)	0	8	4	0	8

Graph

STUDENT COMPLETES



Zeros

GIVEN TO STUDENT

$$x = -2 \text{ and } x = 1$$

Multiplicity and intercept behavior

GIVEN TO STUDENT

$x = -2$ has multiplicity 1; $x = 1$ has multiplicity 2.

End behavior

GIVEN TO STUDENT

As $x \rightarrow -\infty$, $h(x) \rightarrow -\infty$. As $x \rightarrow \infty$, $h(x) \rightarrow \infty$.

Verbal description

STUDENT COMPLETES

A positive cubic. It crosses at -2 , touches and turns at 1 , and has y -intercept 4 .

Fast consistency check. Degree = 3. Leading coefficient = 2. The constant term and the factored form both give $h(0) = 4$.

Why this is the only answer. The multiplicities add to $1 + 2 = 3$, which matches the given degree, so no extra factor can hide in the function. $h(0) = 4$ gives $4 = k(2)(1)$, so $k = 2$.

Function D: Completed Organizer

The standard form hides the zeros. The graph gives you a place to start factoring.

Standard form

GIVEN TO STUDENT

$$p(x) = -x^4 + 5x^2 - 4$$

Zeros

STUDENT COMPLETES

$$x = -2, -1, 1, \text{ and } 2$$

Factored form

STUDENT COMPLETES

$$p(x) = -(x + 2)(x + 1)(x - 1)(x - 2)$$

Multiplicity and intercept behavior

STUDENT COMPLETES

All four zeros have multiplicity 1. The graph crosses at each one.

Table

STUDENT COMPLETES

x	-3	-2	-1	0	1	2	3
p(x)	-40	0	0	-4	0	0	-40

$(-3, -40)$ and $(3, -40)$ sit below the printed grid. Record them in the table only.

End behavior

STUDENT COMPLETES

As $x \rightarrow -\infty$, $p(x) \rightarrow -\infty$. As $x \rightarrow \infty$, $p(x) \rightarrow -\infty$.

Verbal description

STUDENT COMPLETES

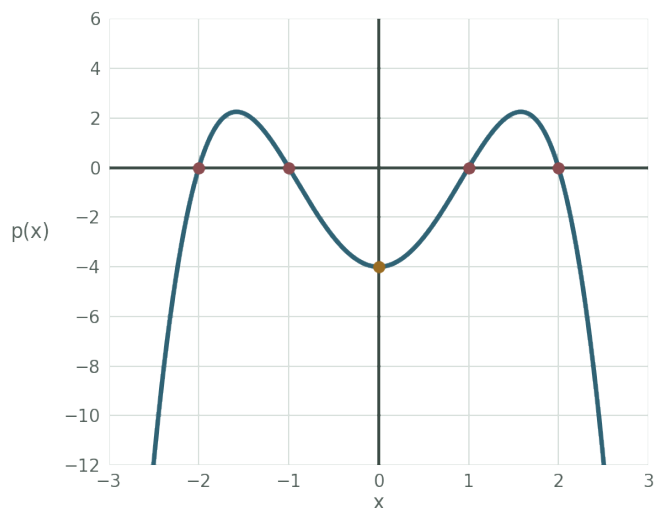
A negative quartic with four simple zeros at -2 , -1 , 1 , and 2 . Both tails fall.

Fast consistency check. Degree = 4. Leading coefficient = -1 . The constant term and the factored form both give $p(0) = -4$.

Why this is the only answer. The standard form is supplied, so the factorization is a fact to be found rather than a choice to be made.

Graph

GIVEN TO STUDENT



Function E: Completed Organizer

The factors show the intercept behavior. Use the table to check the height and symmetry.

Standard form

STUDENT COMPLETES

$$q(x) = x^4 - 2x^3 - 3x^2 + 4x + 4$$

Factored form

GIVEN TO STUDENT

$$q(x) = (x + 1)^2(x - 2)^2$$

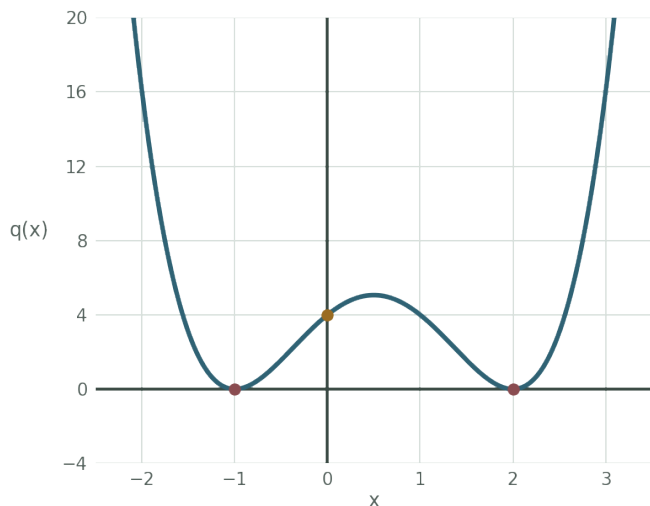
Table

GIVEN TO STUDENT

x	-2	-1	0	1	2	3
q(x)	16	0	4	4	0	16

Graph

STUDENT COMPLETES



Zeros

STUDENT COMPLETES

$$x = -1 \text{ and } x = 2$$

Multiplicity and intercept behavior

STUDENT COMPLETES

Both zeros have multiplicity 2. The graph touches and turns at each one.

End behavior

STUDENT COMPLETES

As $x \rightarrow -\infty$, $q(x) \rightarrow \infty$. As $x \rightarrow \infty$, $q(x) \rightarrow \infty$.

Verbal description

STUDENT COMPLETES

A positive quartic that touches at -1 and 2 . Both tails rise, and the y-intercept is 4 .

Fast consistency check. Degree = 4. Leading coefficient = 1. The constant term and the factored form both give $q(0) = 4$.

Why this is the only answer. The factored form is supplied, so expanding it is the only route to the standard form.

Function F: Completed Organizer

Turn the description into factors first. The y-intercept determines the leading coefficient.

Given clue: $r(0) = -2$

Standard form

STUDENT COMPLETES

$$r(x) = (1/2)x^5 + (1/2)x^4 - (5/2)x^3 - (1/2)x^2 + 4x - 2$$

Factored form

STUDENT COMPLETES

$$r(x) = (1/2) \cdot (x + 2)^2(x - 1)^3$$

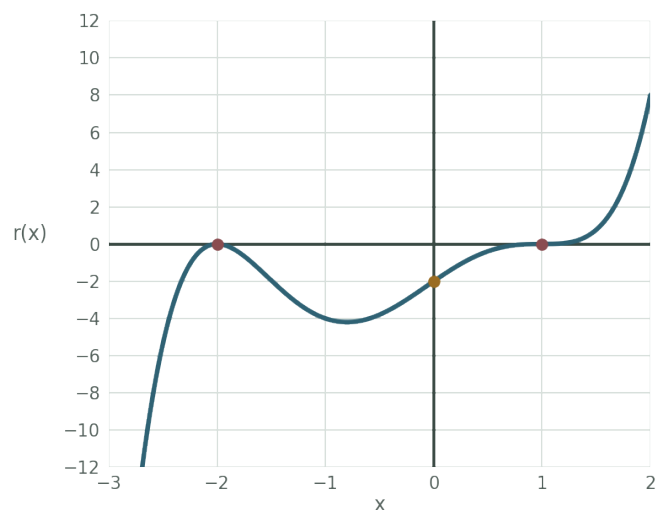
Table

STUDENT COMPLETES

x	-2	-1	0	1	2
r(x)	0	-4	-2	0	8

Graph

STUDENT COMPLETES



Zeros

STUDENT COMPLETES

$$x = -2 \text{ and } x = 1$$

Multiplicity and intercept behavior

STUDENT COMPLETES

$x = -2$ has multiplicity 2; $x = 1$ has multiplicity 3.

End behavior

STUDENT COMPLETES

As $x \rightarrow -\infty$, $r(x) \rightarrow -\infty$. As $x \rightarrow \infty$, $r(x) \rightarrow \infty$.

Verbal description

GIVEN TO STUDENT

A positive fifth-degree function. It touches at -2 , crosses with flattening at 1 , and passes through $(0, -2)$.

Fast consistency check. Degree = 5. Leading coefficient = $1/2$. The constant term and the factored form both give $r(0) = -2$.

Why this is the only answer. "Fifth-degree" fixes the total. Flattening at a crossing means an odd multiplicity of at least 3, so $x = 1$ takes 3 and the touch at -2 takes the remaining 2. $r(0) = -2$ gives $-2 = k(4)(-1)$, so $k = 1/2$.

Function G: Completed Organizer

Build $k(x - r)^m$ from the structure, then use the point to solve for k .

Given clue: Degree 5

Given clue: $s(0) = -4$

Standard form

STUDENT COMPLETES

$$s(x) = -x^5 + x^4 + 5x^3 - x^2 - 8x - 4$$

Factored form

STUDENT COMPLETES

$$s(x) = -(x + 1)^3(x - 2)^2$$

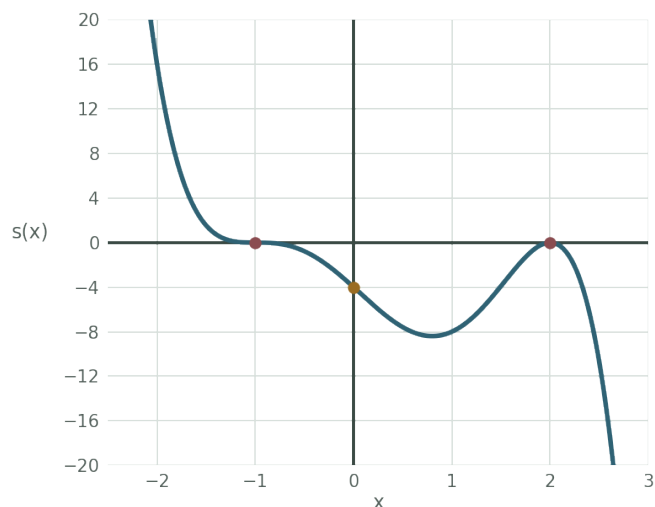
Table

STUDENT COMPLETES

x	-2	-1	0	1	2
s(x)	16	0	-4	-8	0

Graph

STUDENT COMPLETES



Zeros

GIVEN TO STUDENT

$$x = -1 \text{ and } x = 2$$

Multiplicity and intercept behavior

GIVEN TO STUDENT

$x = -1$ has multiplicity 3; $x = 2$ has multiplicity 2.

End behavior

GIVEN TO STUDENT

As $x \rightarrow -\infty$, $s(x) \rightarrow \infty$. As $x \rightarrow \infty$, $s(x) \rightarrow -\infty$.

Verbal description

STUDENT COMPLETES

A negative fifth-degree function. It crosses with flattening at -1 , touches at 2 , rises left, and falls right.

Fast consistency check. Degree = 5. Leading coefficient = -1 . The constant term and the factored form both give $s(0) = -4$.

Why this is the only answer. The multiplicities add to $3 + 2 = 5$, which matches the given degree, so no extra factor fits. $s(0) = -4$ gives $-4 = k(1)(4)$, so $k = -1$.

Function H: Completed Organizer

Only the factored form is supplied. Complete all seven remaining representations.

Standard form

STUDENT COMPLETES

$$t(x) = (1/2)x^4 + (3/2)x^3 - (3/2)x^2 - (11/2)x - 3$$

Factored form

GIVEN TO STUDENT

$$t(x) = (1/2) \cdot (x + 3)(x + 1)^2(x - 2)$$

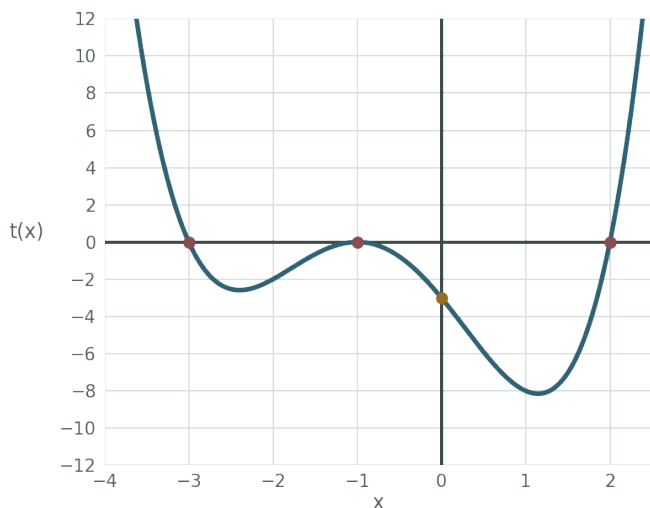
Table

STUDENT COMPLETES

x	-3	-2	-1	0	1	2
t(x)	0	-2	0	-3	-8	0

Graph

STUDENT COMPLETES



Zeros

STUDENT COMPLETES

$x = -3, -1$, and 2

Multiplicity and intercept behavior

STUDENT COMPLETES

$x = -3$ and $x = 2$ have multiplicity 1; $x = -1$ has multiplicity 2.

End behavior

STUDENT COMPLETES

As $x \rightarrow -\infty$, $t(x) \rightarrow \infty$. As $x \rightarrow \infty$, $t(x) \rightarrow \infty$.

Verbal description

STUDENT COMPLETES

A positive quartic. It crosses at -3 and 2 , touches at -1 , passes through $(0, -3)$, and has both tails rising.

Fast consistency check. Degree = 4. Leading coefficient = $1/2$. The constant term and the factored form both give $t(0) = -3$.

Why this is the only answer. The factored form is supplied, so every other box follows from expanding it and reading the factors.