Should I Use Long Division or Synthetic Division?

One question decides it. A second one handles the awkward case.

START: divide P(x) by D(x)

QUESTION 1

Is the divisor a single linear expression — degree 1, written $x - c$ or $ax - b$?

Degree 1 means the highest power of x in the divisor is 1.

NO →

USE LONG DIVISION

The general method. It handles a divisor of any degree, and it is the only one that does.

Divisors that land here

$x^2 + 1$ $\quad x^2 - 3x + 2$ $\quad x^3 - 8$

YES →

QUESTION 2

Is the leading coefficient 1 — the divisor $x - c$ rather than $ax - b$?

YES →

SYNTHETIC DIVISION

Divide by $x - c$ using c. Watch the sign: $x + 3$ means $c = -3$.

$x - 5 \quad x + 3$

NO →

SYNTHETIC, THEN ONE FIX

Use $c = b/a$, then divide the whole quotient row by a.

$2x - 1 \quad 3x + 4$

THE SAME PROBLEM, BOTH WAYS

$$(2x^3 + 3x^2 - 11x - 6) \div (x - 2) = 2x^2 + 7x + 3, \text{ remainder } 0$$

Long division

Subtract a multiple of the divisor at each step until the degree drops below the divisor's.

first the x^2 term, then x, then the constant

Synthetic division

Bring down 2, multiply by 2, add. Repeat. The last number is the remainder.

$2 \mid \; 2 \quad 7 \quad 3 \; \mid \; 0$

BEFORE YOU START, AND AFTER YOU FINISH

Write every missing power as a zero placeholder: $x^3 - 8$ is $x^3 + 0x^2 + 0x - 8$.

Then check: divisor × quotient + remainder should rebuild the original polynomial.

A FREE BONUS FROM SYNTHETIC DIVISION

That last number is P(c). Dividing by $x - 2$ and evaluating P(2) give the same value — which is the Remainder Theorem, and why a remainder of 0 means $x - c$ is a factor.

Polynomial Long Division Worksheet

12 questions · Algebra I and II · answer key available separately

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HOW TO USE THIS PAGE

Long division is the general method. It works no matter what the divisor looks like.

Write the dividend in standard form first, and put a zero placeholder in for every missing power.

Stop when the degree of what is left is smaller than the degree of the divisor. Whatever is left is the remainder.

Check your answer: divisor × quotient + remainder should rebuild the dividend.

SET A · WARM-UP: LINEAR DIVISORS, NO REMAINDER

1. Divide. $(x^2 + 8x + 15) \div (x + 3)$

2. Divide. $(x^3 + 6x^2 + 11x + 6) \div (x + 1)$

3. Divide. $(2x^3 - 3x^2 - 11x + 6) \div (x - 3)$

4. Divide. $(x^3 + 27) \div (x + 3)$

SET B · PLACEHOLDERS AND REMAINDERS

Write every missing power as a zero placeholder before you set up the bracket.

5. Divide. $(x^3 - 4x^2 + 2) \div (x - 1)$

6. Divide. $(2x^4 + x^3 - 3x + 5) \div (x + 2)$

7. Divide. $(4x^3 - 7x + 1) \div (2x - 3)$

8. Divide. $(x^4 - 1) \div (x - 1)$

SET C · DIVISORS THAT ARE NOT LINEAR

Synthetic division does not reach these. Long division does.

9. Divide. $(x^3 + 2x^2 - 5x - 6) \div (x^2 - 1)$

10. Divide. $(x^4 - 3x^2 + 2) \div (x^2 - 2)$

11. Divide. $(2x^4 + 3x^3 - x + 4) \div (x^2 + x - 1)$

12. Divide. $(x^5 - 32) \div (x - 2)$

Answers with fully worked steps are in the polynomial division answer key at burketutoringinfremont.com/polynomial-division-worksheet/

Synthetic Division Worksheet

12 questions · Algebra I and II · answer key available separately

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HOW TO USE THIS PAGE

Synthetic division is a shortcut, and it only applies when the divisor is linear.

Dividing by $x - c$ means you use c. Dividing by $x + 4$ means $c = -4$, not 4.

The top row needs a coefficient for every power from the highest down to the constant. Missing power, write 0.

The last number in the bottom row is the remainder. Everything before it is the quotient, one degree lower than the dividend.

SET A · MONIC LINEAR DIVISORS

Watch the sign. Dividing by $x + 4$ means $c = -4$.

1. Divide. $(x^3 - 2x^2 - 5x + 6) \div (x - 1)$

2. Divide. $(x^3 + 2x^2 - 5x + 12) \div (x + 4)$

3. Divide. $(2x^3 + 3x^2 - 8x + 3) \div (x - 1)$

4. Divide. $(x^3 - 5x^2 + 2x + 8) \div (x - 2)$

SET B · MISSING TERMS AND REMAINDERS

Every power from the highest down to the constant needs a coefficient in the top row.

5. Divide. $(x^4 - 5x^2 + 4) \div (x - 1)$

6. Divide. $(3x^3 + 2x - 7) \div (x + 1)$

7. Divide. $(x^5 - 1) \div (x - 1)$

8. Divide. $(2x^4 - 3x^3 + x - 5) \div (x - 3)$

SET C · DIVISORS OF THE FORM AX − B

Use c = b/a, then divide the whole quotient row by a. The remainder does not get divided.

9. Divide. $(2x^3 + 7x^2 - 4x - 15) \div (2x + 3)$

10. Divide. $(3x^3 - 4x^2 - 5x + 2) \div (3x - 1)$

11. Divide. $(x^3 - 3x^2 + 4) \div (x + 1)$

12. Divide. $(x^4 + 2x^3 - 7x^2 - 8x + 12) \div (x - 1)$

Answers with the full synthetic rows are in the polynomial division answer key at burketutoringinfremont.com/polynomial-division-worksheet/

Long Division or Synthetic Division? Mixed Practice

12 questions · choose the method, say why, then divide

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CHOOSE BEFORE YOU COMPUTE

For each problem, write LONG or SYNTHETIC in the margin first, then a short reason.

The only question that matters: is the divisor linear? If it is not, synthetic division cannot reach it.

If the divisor is $ax - b$ with $a \neq 1$, synthetic still works — use $c = b/a$, then divide the quotient row by a.

Some of these look harder than they are. Check the divisor before you decide.

SET A · NAME THE METHOD, THEN DIVIDE

Write LONG or SYNTHETIC in the box before you start, and one sentence saying why.

1. $(x^3 - x^2 - 14x + 24) \div (x - 3)$ **method:** ____________

2. $(x^4 + x^3 - 11x^2 - 9x + 18) \div (x^2 - 9)$ **method:** ____________

3. $(3x^3 + 2x^2 - 19x + 6) \div (x + 3)$ **method:** ____________

4. $(x^4 - 6x^2 + 8) \div (x^2 - 4)$ **method:** ____________

SET B · THE AWKWARD DIVISORS

5. $(6x^3 + 5x^2 - 21x + 10) \div (3x - 2)$ **method:** ____________

6. $(x^3 + 5x^2 + 2x - 8) \div (x + 2)$ **method:** ____________

7. $(2x^5 - 3x^2 + 7) \div (x - 1)$ **method:** ____________

8. $(x^4 + 2x^3 + 3x^2 + 2x + 1) \div (x^2 + x + 1)$ **method:** ____________

SET C · MIXED PRACTICE

9. $(4x^3 - 9x) \div (2x - 3)$ **method:** ____________

10. $(x^3 - 8) \div (x - 2)$ **method:** ____________

11. $(x^4 - 5x^3 + 6x^2 + 4x - 8) \div (x - 2)$ **method:** ____________

12. $(2x^4 + x^3 - 3x^2 + 5x - 1) \div (x^2 - x + 2)$ **method:** ____________

Answers with fully worked steps are in the polynomial division answer key at burketutoringinfremont.com/polynomial-division-worksheet/

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THE TWO STATEMENTS YOU ARE USING

Remainder Theorem. When P(x) is divided by x − c, the remainder is P(c). So you can find a remainder by evaluating, with no division at all.

Factor Theorem. (x − c) is a factor of P(x) exactly when P(c) = 0. A remainder of zero is the whole test.

For a divisor like 2x − 1, the matching value is c = 1/2, because that is where the divisor equals zero.

SET A · REMAINDER THEOREM

Evaluate, or run one line of synthetic division. Both give the same number.

1. $P(x) = x^3 - 5x^2 + 3x + 7$. Find $P(2)$.

2. $P(x) = 2x^4 - x^3 + 3x - 6$. Find $P(-1)$.

3. Find the remainder when $x^4 - 3x^3 + 2x - 5$ is divided by $x + 2$.

4. Find the remainder when $3x^3 + 2x^2 - x + 4$ is divided by $2x - 1$.

SET B · FACTOR THEOREM: TEST THE CANDIDATE

A factor leaves a remainder of zero. Nothing else counts as a factor.

5. Is $(x - 3)$ a factor of $x^3 - 2x^2 - 5x + 6$?

6. Is $(x + 1)$ a factor of $2x^3 + 3x^2 - 4x - 5$?

7. Is $(x - 2)$ a factor of $x^4 - 3x^2 + 4x - 8$?

8. Which of $x - 1$, $x + 2$, $x - 4$ are factors of $x^3 - 3x^2 - 6x + 8$?

SET C · SOLVING FOR AN UNKNOWN COEFFICIENT

Set the remainder equal to what the question requires, then solve for the letter.

9. Find k so that $(x - 2)$ is a factor of $x^3 + kx^2 - 7x + 2$.

10. Find k so that the remainder is 4 when $x^3 - 4x^2 + kx + 2$ is divided by $x - 2$.

11. Find a so that $(x + 1)$ is a factor of $ax^3 + 5x^2 - 2x - 8$.

12. $P(x) = x^3 + bx + 6$ leaves a remainder of 12 when divided by $x - 3$. Find b.

SET D · FINDING THE REMAINING ZEROS

Divide out the factor you were given, then factor or solve the quotient.

13. $x = -2$ is a zero of $P(x) = x^3 + x^2 - 4x - 4$. Find the remaining zeros.

14. $(x - 1)$ is a factor of $P(x) = 2x^3 + 3x^2 - 8x + 3$. Factor $P(x)$ completely and list all zeros.

15. $x = 3$ is a zero of $P(x) = 2x^3 - 5x^2 - 4x + 3$. Factor completely and list all zeros.

16. Factor $x^4 - 5x^2 + 4$ into linear factors and list every zero.

Worked solutions are in the polynomial division answer key at burketutoringinfremont.com/polynomial-division-worksheet/

Polynomial Division — Fully Worked Solutions

Every step for all 52 questions across the four worksheets

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Each solution shows the working, not just the result. Steps are written the way a student should write them, so the page can be handed out as a self-check or used for grading without the worksheets in hand.

WORKSHEET 1 · POLYNOMIAL LONG DIVISION

1. $(x^2 + 8x + 15) \div (x + 3)$

$x \cdot (x + 3) = x^2 + 3x \rightarrow$ subtract: $5x + 15$

$5 \cdot (x + 3) = 5x + 15 \rightarrow$ subtract: 0

Quotient $x + 5$, remainder 0

2. $(x^3 + 6x^2 + 11x + 6) \div (x + 1)$

$x^2 \cdot (x + 1) = x^3 + x^2 \rightarrow$ subtract: $5x^2 + 11x + 6$

$5x \cdot (x + 1) = 5x^2 + 5x \rightarrow$ subtract: $6x + 6$

$6 \cdot (x + 1) = 6x + 6 \rightarrow$ subtract: 0

Quotient $x^2 + 5x + 6$, remainder 0

3. $(2x^3 - 3x^2 - 11x + 6) \div (x - 3)$

$2x^2 \cdot (x - 3) = 2x^3 - 6x^2 \rightarrow$ subtract: $3x^2 - 11x + 6$

$3x \cdot (x - 3) = 3x^2 - 9x \rightarrow$ subtract: $-2x + 6$

$-2 \cdot (x - 3) = -2x + 6 \rightarrow$ subtract: 0

Quotient $2x^2 + 3x - 2$, remainder 0

4. $(x^3 + 27) \div (x + 3)$

$x^2 \cdot (x + 3) = x^3 + 3x^2 \rightarrow$ subtract: $-3x^2 + 27$

$-3x \cdot (x + 3) = -3x^2 - 9x \rightarrow$ subtract: $9x + 27$

$9 \cdot (x + 3) = 9x + 27 \rightarrow$ subtract: 0

Quotient $x^2 - 3x + 9$, remainder 0

5. $(x^3 - 4x^2 + 2) \div (x - 1)$

$x^2 \cdot (x - 1) = x^3 - x^2 \rightarrow$ subtract: $-3x^2 + 2$

$-3x \cdot (x - 1) = -3x^2 + 3x \rightarrow$ subtract: $-3x + 2$

$-3 \cdot (x - 1) = -3x + 3 \rightarrow$ subtract: -1

Quotient $x^2 - 3x - 3$, remainder −1

6. $(2x^4 + x^3 - 3x + 5) \div (x + 2)$

$2x^3 \cdot (x + 2) = 2x^4 + 4x^3 \rightarrow$ subtract: $-3x^3 - 3x + 5$

$-3x^2 \cdot (x + 2) = -3x^3 - 6x^2 \rightarrow$ subtract: $6x^2 - 3x + 5$

$6x \cdot (x + 2) = 6x^2 + 12x \rightarrow$ subtract: $-15x + 5$

$-15 \cdot (x + 2) = -15x - 30 \rightarrow$ subtract: 35

Quotient $2x^3 - 3x^2 + 6x - 15$, remainder 35

7. $(4x^3 - 7x + 1) \div (2x - 3)$

$2x^2 \cdot (2x - 3) = 4x^3 - 6x^2 \rightarrow$ subtract: $6x^2 - 7x + 1$

$3x \cdot (2x - 3) = 6x^2 - 9x \rightarrow$ subtract: $2x + 1$

$1 \cdot (2x - 3) = 2x - 3 \rightarrow$ subtract: 4

Quotient $2x^2 + 3x + 1$, remainder 4

8. $(x^4 - 1) \div (x - 1)$

$x^3 \cdot (x - 1) = x^4 - x^3 \rightarrow$ subtract: $x^3 - 1$

$x^2 \cdot (x - 1) = x^3 - x^2 \rightarrow$ subtract: $x^2 - 1$

$x \cdot (x - 1) = x^2 - x \rightarrow$ subtract: $x - 1$

$1 \cdot (x - 1) = x - 1 \rightarrow$ subtract: 0

Quotient $x^3 + x^2 + x + 1$, remainder 0

9. $(x^3 + 2x^2 - 5x - 6) \div (x^2 - 1)$

$x \cdot (x^2 - 1) = x^3 - x \rightarrow$ subtract: $2x^2 - 4x - 6$

$2 \cdot (x^2 - 1) = 2x^2 - 2 \rightarrow$ subtract: $-4x - 4$

Quotient $x + 2$, remainder $-4x - 4$

10. $(x^4 - 3x^2 + 2) \div (x^2 - 2)$

$x^2 \cdot (x^2 - 2) = x^4 - 2x^2 \rightarrow$ subtract: $-x^2 + 2$

$-1 \cdot (x^2 - 2) = -x^2 + 2 \rightarrow$ subtract: 0

Quotient $x^2 - 1$, remainder 0

11. $(2x^4 + 3x^3 - x + 4) \div (x^2 + x - 1)$

$2x^2 \cdot (x^2 + x - 1) = 2x^4 + 2x^3 - 2x^2 \rightarrow$ subtract: $x^3 + 2x^2 - x + 4$

$x \cdot (x^2 + x - 1) = x^3 + x^2 - x \rightarrow$ subtract: $x^2 + 4$

$1 \cdot (x^2 + x - 1) = x^2 + x - 1 \rightarrow$ subtract: $-x + 5$

Quotient $2x^2 + x + 1$, remainder $-x + 5$

12. $(x^5 - 32) \div (x - 2)$

$x^4 \cdot (x - 2) = x^5 - 2x^4 \rightarrow$ subtract: $2x^4 - 32$

$2x^3 \cdot (x - 2) = 2x^4 - 4x^3 \rightarrow$ subtract: $4x^3 - 32$

$4x^2 \cdot (x - 2) = 4x^3 - 8x^2 \rightarrow$ subtract: $8x^2 - 32$

$8x \cdot (x - 2) = 8x^2 - 16x \rightarrow$ subtract: $16x - 32$

$16 \cdot (x - 2) = 16x - 32 \rightarrow$ subtract: 0

Quotient $x^4 + 2x^3 + 4x^2 + 8x + 16$, remainder 0

WORKSHEET 2 · SYNTHETIC DIVISION

1. $(x^3 - 2x^2 - 5x + 6) \div (x - 1)$

c = 1 coefficients: 1 −2 −5 6

carry down and add: 1 −1 −6 0 (last entry is the remainder)

Quotient $x^2 - x - 6$, remainder 0

2. $(x^3 + 2x^2 - 5x + 12) \div (x + 4)$

c = −4 coefficients: 1 2 −5 12

carry down and add: 1 −2 3 0 (last entry is the remainder)

Quotient $x^2 - 2x + 3$, remainder 0

3. $(2x^3 + 3x^2 - 8x + 3) \div (x - 1)$

c = 1 coefficients: 2 3 −8 3

carry down and add: 2 5 −3 0 (last entry is the remainder)

Quotient $2x^2 + 5x - 3$, remainder 0

4. $(x^3 - 5x^2 + 2x + 8) \div (x - 2)$

c = 2 coefficients: 1 −5 2 8

carry down and add: 1 −3 −4 0 (last entry is the remainder)

Quotient $x^2 - 3x - 4$, remainder 0

5. $(x^4 - 5x^2 + 4) \div (x - 1)$

c = 1 coefficients: 1 0 −5 0 4

carry down and add: 1 1 −4 −4 0 (last entry is the remainder)

Quotient $x^3 + x^2 - 4x - 4$, remainder 0

6. $(3x^3 + 2x - 7) \div (x + 1)$

c = −1 coefficients: 3 0 2 −7

carry down and add: 3 −3 5 −12 (last entry is the remainder)

Quotient $3x^2 - 3x + 5$, remainder −12

7. $(x^5 - 1) \div (x - 1)$

c = 1 coefficients: 1 0 0 0 0 −1

carry down and add: 1 1 1 1 1 0 (last entry is the remainder)

Quotient $x^4 + x^3 + x^2 + x + 1$, remainder 0

8. $(2x^4 - 3x^3 + x - 5) \div (x - 3)$

c = 3 coefficients: 2 −3 0 1 −5

carry down and add: 2 3 9 28 79 (last entry is the remainder)

Quotient $2x^3 + 3x^2 + 9x + 28$, remainder 79

9. $(2x^3 + 7x^2 - 4x - 15) \div (2x + 3)$

c = −3/2 coefficients: 2 7 −4 −15

carry down and add: 2 4 −10 0 (last entry is the remainder)

The divisor is $2x + 3$, so divide the quotient row by 2. The remainder is not divided.

Quotient $x^2 + 2x - 5$, remainder 0

10. $(3x^3 - 4x^2 - 5x + 2) \div (3x - 1)$

c = 1/3 coefficients: 3 −4 −5 2

carry down and add: 3 −3 −6 0 (last entry is the remainder)

The divisor is $3x - 1$, so divide the quotient row by 3. The remainder is not divided.

Quotient $x^2 - x - 2$, remainder 0

11. $(x^3 - 3x^2 + 4) \div (x + 1)$

c = −1 coefficients: 1 −3 0 4

carry down and add: 1 −4 4 0 (last entry is the remainder)

Quotient $x^2 - 4x + 4$, remainder 0

12. $(x^4 + 2x^3 - 7x^2 - 8x + 12) \div (x - 1)$

c = 1 coefficients: 1 2 −7 −8 12

carry down and add: 1 3 −4 −12 0 (last entry is the remainder)

Quotient $x^3 + 3x^2 - 4x - 12$, remainder 0

WORKSHEET 3 · CHOOSING A METHOD

1. $(x^3 - x^2 - 14x + 24) \div (x - 3)$

Divisor is linear and monic, so synthetic division is quickest.

c = 3 coefficients: 1 −1 −14 24

carry down and add: 1 2 −8 0 (last entry is the remainder)

Quotient $x^2 + 2x - 8$, remainder 0

2. $(x^4 + x^3 - 11x^2 - 9x + 18) \div (x^2 - 9)$

Divisor is not linear, so long division is the only option.

$x^2 \cdot (x^2 - 9) = x^4 - 9x^2 \rightarrow$ subtract: $x^3 - 2x^2 - 9x + 18$

$x \cdot (x^2 - 9) = x^3 - 9x \rightarrow$ subtract: $-2x^2 + 18$

$-2 \cdot (x^2 - 9) = -2x^2 + 18 \rightarrow$ subtract: 0

Quotient $x^2 + x - 2$, remainder 0

3. $(3x^3 + 2x^2 - 19x + 6) \div (x + 3)$

Divisor is linear and monic, so synthetic division is quickest.

$c = -3$ coefficients: 3 2 −19 6

carry down and add: 3 −7 2 0 (last entry is the remainder)

Quotient $3x^2 - 7x + 2$, remainder 0

4. $(x^4 - 6x^2 + 8) \div (x^2 - 4)$

Divisor is not linear, so long division is the only option.

$x^2 \cdot (x^2 - 4) = x^4 - 4x^2 \rightarrow$ subtract: $-2x^2 + 8$

$-2 \cdot (x^2 - 4) = -2x^2 + 8 \rightarrow$ subtract: 0

Quotient $x^2 - 2$, remainder 0

5. $(6x^3 + 5x^2 - 21x + 10) \div (3x - 2)$

Divisor is linear but not monic. Synthetic division with $c = 2/3$, then divide the quotient row by 3.

$c = 2/3$ coefficients: 6 5 −21 10

carry down and add: 6 9 −15 0 (last entry is the remainder)

Quotient row ÷ 3 gives $2x^2 + 3x - 5$.

Quotient $2x^2 + 3x - 5$, remainder 0

6. $(x^3 + 5x^2 + 2x - 8) \div (x + 2)$

Divisor is linear and monic, so synthetic division is quickest.

$c = -2$ coefficients: 1 5 2 −8

carry down and add: 1 3 −4 0 (last entry is the remainder)

Quotient $x^2 + 3x - 4$, remainder 0

7. $(2x^5 - 3x^2 + 7) \div (x - 1)$

Divisor is linear and monic, so synthetic division is quickest.

$c = 1$ coefficients: 2 0 0 −3 0 7

carry down and add: 2 2 2 −1 −1 6 (last entry is the remainder)

Quotient $2x^4 + 2x^3 + 2x^2 - x - 1$, remainder 6

8. $(x^4 + 2x^3 + 3x^2 + 2x + 1) \div (x^2 + x + 1)$

Divisor is not linear, so long division is the only option.

$x^2 \cdot (x^2 + x + 1) = x^4 + x^3 + x^2 \rightarrow$ subtract: $x^3 + 2x^2 + 2x + 1$

$x \cdot (x^2 + x + 1) = x^3 + x^2 + x \rightarrow$ subtract: $x^2 + x + 1$

$1 \cdot (x^2 + x + 1) = x^2 + x + 1 \rightarrow$ subtract: 0

Quotient $x^2 + x + 1$, remainder 0

9. $(4x^3 - 9x) \div (2x - 3)$

Divisor is linear but not monic. Synthetic division with c = 3/2, then divide the quotient row by 2.

c = 3/2 coefficients: 4 0 −9 0

carry down and add: 4 6 0 0 (last entry is the remainder)

Quotient row ÷ 2 gives $2x^2 + 3x$.

Quotient $2x^2 + 3x$, remainder 0

10. $(x^3 - 8) \div (x - 2)$

Divisor is linear and monic, so synthetic division is quickest.

c = 2 coefficients: 1 0 0 −8

carry down and add: 1 2 4 0 (last entry is the remainder)

Quotient $x^2 + 2x + 4$, remainder 0

11. $(x^4 - 5x^3 + 6x^2 + 4x - 8) \div (x - 2)$

Divisor is linear and monic, so synthetic division is quickest.

c = 2 coefficients: 1 −5 6 4 −8

carry down and add: 1 −3 0 4 0 (last entry is the remainder)

Quotient $x^3 - 3x^2 + 4$, remainder 0

12. $(2x^4 + x^3 - 3x^2 + 5x - 1) \div (x^2 - x + 2)$

Divisor is not linear, so long division is the only option.

$2x^2 \cdot (x^2 - x + 2) = 2x^4 - 2x^3 + 4x^2 \rightarrow$ subtract: $3x^3 - 7x^2 + 5x - 1$

$3x \cdot (x^2 - x + 2) = 3x^3 - 3x^2 + 6x \rightarrow$ subtract: $-4x^2 - x - 1$

$-4 \cdot (x^2 - x + 2) = -4x^2 + 4x - 8 \rightarrow$ subtract: $-5x + 7$

Quotient $2x^2 + 3x - 4$, remainder $-5x + 7$

WORKSHEET 4 · REMAINDER AND FACTOR THEOREMS

1. $P(x) = x^3 - 5x^2 + 3x + 7$. Find P(2).

$P(2) = 8 - 20 + 6 + 7 = 1$

2. $P(x) = 2x^4 - x^3 + 3x - 6$. Find P(−1).

$P(-1) = 2 + 1 - 3 - 6 = -6$

3. Find the remainder when $x^4 - 3x^3 + 2x - 5$ is divided by $x + 2$.

$P(-2) = 16 + 24 - 4 - 5 = 31$, so the remainder is 31

4. Find the remainder when $3x^3 + 2x^2 - x + 4$ is divided by $2x - 1$.

$c = 1/2$. $P(1/2) = 3/8 + 1/2 - 1/2 + 4 = 35/8$, so the remainder is 35/8

5. Is $(x - 3)$ a factor of $x^3 - 2x^2 - 5x + 6$?

$P(3) = 27 - 18 - 15 + 6 = 0$. Yes, $(x - 3)$ is a factor.

6. Is $(x + 1)$ a factor of $2x^3 + 3x^2 - 4x - 5$?

$P(-1) = -2 + 3 + 4 - 5 = 0$. Yes, $(x + 1)$ is a factor.

7. Is $(x - 2)$ a factor of $x^4 - 3x^2 + 4x - 8$?

$P(2) = 16 - 12 + 8 - 8 = 4$. No. The remainder is 4, not 0.

8. Which of $x - 1$, $x + 2$, $x - 4$ are factors of $x^3 - 3x^2 - 6x + 8$?

$P(1) = 0$ and $P(-2) = 0$, so $x - 1$ and $x + 2$ are factors. $P(4) = 0$ as well, so all three are: $x^3 - 3x^2 - 6x + 8 = (x - 1)(x + 2)(x - 4)$.

9. Find k so that $(x - 2)$ is a factor of $x^3 + kx^2 - 7x + 2$.

$P(2) = 8 + 4k - 14 + 2 = 4k - 4$. Set $4k - 4 = 0$, so $k = 1$.

10. Find k so that the remainder is 4 when $x^3 - 4x^2 + kx + 2$ is divided by $x - 2$.

$P(2) = 8 - 16 + 2k + 2 = 2k - 6$. Set $2k - 6 = 4$, so $k = 5$.

11. Find a so that $(x + 1)$ is a factor of $ax^3 + 5x^2 - 2x - 8$.

$P(-1) = -a + 5 + 2 - 8 = -a - 1$. Set $-a - 1 = 0$, so $a = -1$.

12. $P(x) = x^3 + bx + 6$ leaves a remainder of 12 when divided by $x - 3$. Find b.

$P(3) = 27 + 3b + 6 = 3b + 33$. Set $3b + 33 = 12$, so $b = -7$.

13. $x = -2$ is a zero of $P(x) = x^3 + x^2 - 4x - 4$. Find the remaining zeros.

Synthetic division by $x + 2$ gives $x^2 - x - 2 = (x - 2)(x + 1)$. Remaining zeros: $x = 2$ and $x = -1$.

14. $(x - 1)$ is a factor of $P(x) = 2x^3 + 3x^2 - 8x + 3$. Factor P(x) completely and list all zeros.

Quotient $2x^2 + 5x - 3 = (2x - 1)(x + 3)$, so $P(x) = (x - 1)(2x - 1)(x + 3)$. Zeros: $x = 1$, $x = 1/2$, $x = -3$.

15. $x = 3$ is a zero of $P(x) = 2x^3 - 5x^2 - 4x + 3$. Factor completely and list all zeros.

Quotient $2x^2 + x - 1 = (2x - 1)(x + 1)$, so $P(x) = (x - 3)(2x - 1)(x + 1)$. Zeros: $x = 3$, $x = 1/2$, $x = -1$.

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16. Factor $x^4 - 5x^2 + 4$ into linear factors and list every zero.

$(x^2 - 1)(x^2 - 4) = (x - 1)(x + 1)(x - 2)(x + 2)$. Zeros: $x = 1, -1, 2, -2$.

WHERE THE POINTS ACTUALLY GO MISSING

The dropped placeholder. A missing power that never got written as 0 shifts every column after it. Most wrong answers on this unit start here.

The sign on c. Dividing by $x + 4$ means $c = -4$. Students who write 4 get a clean-looking answer that is wrong from the first product.

Subtracting only the first term. In long division the whole product gets subtracted, not just the leading term.

Forgetting to divide the quotient row. After synthetic division by $ax - b$, the quotient row still has to be divided by a. The remainder does not.

Stopping too early. Keep going until what is left has a lower degree than the divisor.

The fastest check is multiplication: divisor × quotient + remainder should rebuild the dividend exactly.