What the project is

Students cut four equal squares from the corners of an 8.5 in by 11 in sheet, fold up the flaps, and end up with an open-top tray. The volume is a cubic in the size of the corner cut. They build that function, restrict its domain, tabulate it, graph it, estimate the maximum, and then argue for one specific cut size against a written design brief.

The move that makes this project worth running

The brief includes a shelf the tray has to fit. That rule pushes the mathematical maximum **out of bounds**. Students who stop at “find the peak” get an answer that is genuinely wrong, and the ones who read the brief get a different number. Part G stops being a paragraph of filler and starts being an argument.

Logistics

Course	Algebra 2, or a strong Algebra 1 class after polynomial multiplication. Also works as precalculus review.
Time	Two 50-minute periods plus the build, or one period with Parts C through E set as homework.
Grouping	Pairs for Parts A through F, individual writing for Part G. The written defence is where copying shows up, so keep it individual.
Materials	Card stock or manila folders cut to 8.5 by 11, rulers with eighths, scissors, tape, and rice or dry beans if you want the volume check at the end.
Technology	Optional. Every number in the packet can be reached with a four-function calculator. Graphing software makes Part E faster but removes most of the thinking.

Standards

Standard	Where it shows up
HSA.CED.A.2	Part A, writing V(x) from the picture.
HSA.CED.A.3	Parts B and F, representing the brief as constraints and judging which solutions are viable. This is the spine of the project.
HSA.SSE.A.2	Part A, moving between factored and standard form.
HSA.APR.B.3	Part A question 5, zeros of the cubic and what each one means physically.
HSF.IF.B.4	Part D, reading maximum and end behaviour off the graph.
HSF.IF.B.5	Part B, relating the domain to the quantity it describes.
HSF.IF.C.7c	Part D, graphing a polynomial from zeros and a table.
HSG.MG.A.3	Parts F and G, applying geometry to a design problem with physical constraints.

Standard	Where it shows up
MP.3 and MP.4	Part G, building and defending a model.

The numbers, so you can grade quickly

Every value below was checked with a computer algebra system before publication.

The model

$V(x) = x(11 - 2x)(8.5 - 2x) = 4x^3 - 39x^2 + 93.5x$

Zeros at $x = 0$, $x = 4.25$ and $x = 5.5$. Only the first two are reachable on a real sheet.

Domain from the sheet alone: $0 < x < 4.25$.

Feasible interval once the brief is applied: $1.75 \le x < 4.25$.

Part C table

x (in)	Base (in)	Volume (in³)	Meets R2 and R3?
0.25	10.50 × 8.00	21.00	no
0.50	10.00 × 7.50	37.50	no
0.75	9.50 × 7.00	49.88	no
1.00	9.00 × 6.50	58.50	no
1.25	8.50 × 6.00	63.75	no
1.50	8.00 × 5.50	66.00	no
1.75	7.50 × 5.00	65.63	**yes**
2.00	7.00 × 4.50	63.00	yes
2.25	6.50 × 4.00	58.50	yes
2.50	6.00 × 3.50	52.50	yes
3.00	5.00 × 2.50	37.50	yes
3.50	4.00 × 1.50	21.00	yes
4.00	3.00 × 0.50	6.00	yes

Watch for this in question 13

$x = 1.00$ and $x = 2.25$ both give exactly 58.50 in³. One tray is 9 by 6.5 with 1 in walls, the other is 6.5 by 4 with 2.25 in walls. Same volume, very different object. It is the cleanest way into the idea that a horizontal line can cut a cubic more than once.

Answers, Part by Part

Part A

Q	Answer
1	Height x, longer base side $11 - 2x$, shorter base side $8.5 - 2x$.
2	$V(x) = x(11 - 2x)(8.5 - 2x)$.
3	$V(x) = 4x^3 - 39x^2 + 93.5x$.
4	Degree 3, leading coefficient 4. Positive leading coefficient on an odd degree, so the curve falls on the far left and rises on the far right. Neither end matters on a real sheet, which is the point of Part B.
5	$x = 0, 4.25, 5.5$. At $x = 0$ nothing is cut and there is no box. At $x = 4.25$ the short side closes to zero. $x = 5.5$ would need the short side to be cut past zero, so no sheet produces it.

Part B

Q	Answer
6	A flat sheet. No walls, no volume.
7	$8.5 - 2x > 0$, so $x < 4.25$. The short side is what binds, not the long one.
8	$0 < x < 4.25$.
9	$x \geq 1$.
10	Width fit: $8.5 - 2x \leq 5$ gives $x \geq 1.75$. Depth fit: $11 - 2x \leq 8$ gives $x \geq 1.5$. Both have to hold.
11	$1.75 \leq x < 4.25$. The width fit is the binding rule; the 1 in wall rule and the depth fit are already satisfied by everything in that interval.

Parts D and E

Q	Answer
14, 15	P is the peak of the curve at $x \approx 1.585$, $V \approx 66.15$. Q sits at the left end of the feasible interval, $x = 1.75$, $V = 65.625$. P and Q are different points, and noticing that is the whole project.
16	The peak is at $x \approx 1.59$ in with $V \approx 66.15$ in^3. Two decimal places is about as far as a 0.02 table can honestly take them. The exact value is $13/4 - \sqrt{399}/12 \approx 1.5854$.
17	$1.500 \to 66.000$, $1.625 \to 66.117$, $1.750 \to 65.625$, $1.875 \to 64.570$. The best eighth on the whole curve is 1 5/8 in.
18	Outside it. The peak sits at 1.585, and the shelf rule does not allow anything below 1.75. So the design is decided at the boundary rather than at the peak.

Parts F and G

Q	Answer
Design	$x = 1.75$ in (1 3/4 in). Base 7.5 in by 5.0 in, walls 1.75 in, volume 65.625 in^3.
19	65.625 in^3, or 65.63 to two decimals.
20	$66.148 - 65.625 = 0.523$ in^3, about 0.79 percent.
22	The argument has to cover the whole interval, not a list of tested values. The clean version: V has one turning point on $0 < x < 4.25$, at $x \approx 1.585$, and it is a maximum, so V is decreasing for every x above 1.585. The feasible interval starts at 1.75, which is already past the turn. So the largest feasible volume is at the left endpoint.
23	Almost always $x = 1.625$ (66.117 in^3) or $x = 1.5$ (66.000). Those trays are 5.25 in and 5.5 in wide. The shelf opening is 5 in. Both fail R3.
24	Yes, it changes. A 6 in opening gives $8.5 - 2x \le 6$, so $x \ge 1.25$, and the depth rule still forces $x \ge 1.5$. The peak at 1.585 is now inside the feasible interval, so the answer becomes the nearest good eighth, $x = 1.625$ in, $V = 66.117$ in^3.

Where students get stuck

Subtracting one x instead of two

$V = x(11 - x)(8.5 - x)$ is the single most common wrong model. A corner square is removed from *both* ends of every side. Send them back to the flat sheet drawing and have them trace one full edge with a finger, counting the squares they cross.

Using the long side for the domain

Plenty of students write $0 < x < 5.5$. Ask what the tray looks like at $x = 5$ and let them try to draw it.

Confusing the domain of the function with the domain of the model

The cubic itself is defined everywhere. Say the two phrases out loud as different questions, more than once.

Finding the peak and stopping

This is the failure the brief is designed to catch. Do not warn them in advance. Let Part F catch it, then let them fix it.

Rounding to a cut nobody can make

1.59 in is not a mark on a ruler. Rule R4 exists so that rounding becomes a modelling decision rather than a formatting one.

Checking one fit and forgetting the other

The shelf has a width and a depth. Both produce an inequality. Students who check only the width still land on 1.75, so they look right for the wrong reason. Ask for both.

Arguing from a table instead of from the function

“I tested every value” is false when the step is 0.25. Push for a statement about the whole interval: one turning point, so one direction of travel after it.

Reporting the built tray as a failure

The measured volume comes out under the prediction because folds have thickness and card does not crease on a line. That gap is a finding, not an error.

Running it

Two-period version

When	What
Day 1, first 10 min	Hand each pair a sheet and let them cut a tray with any corner size they like. No maths yet. Collect three trays with visibly different shapes and stand them at the front.
Day 1, 30 min	Parts A and B. Stop the class when the first pair writes $11 - x$ and put both versions on the board without saying which is right.
Day 1, last 10 min	Start Part C. Finish it for homework.
Day 2, 25 min	Parts D and E. A quick vote on the best cut before Part F is worth the two minutes it costs.
Day 2, 25 min	Part F, then Part G in silence. The moment a pair discovers 1.625 in does not fit the shelf is the moment the project works.

One-period version

Give out Part A and B completed, keep Parts C through G. That preserves the constraint argument, which is the part worth the time, and drops the algebra the class has probably already done elsewhere.

If it goes fast

Two other resources on this site now build on the same idea. The poster board packet uses a 16 in by 10 in sheet where the maximum lands on a whole number, and the extension questions that go with it include a scaling result and a proof that the best cut on a square sheet is always one sixth of the side. The advanced version uses a 36 in by 24 in sheet whose exact maximum is irrational. None of the three shares its numbers with this letter sheet packet, since each uses its own sheet size, so treat them as a further activity rather than a direct extension of this table.

A note on the build

Do the build. A class that only graphs the function will argue about the third decimal place. A class holding two trays that differ by half a cubic inch argues about whether the difference is worth measuring, which is the better argument.