The brief

The garden club needs seedling trays. You are going to design one, and you only get one sheet of card stock to do it with.

Your job

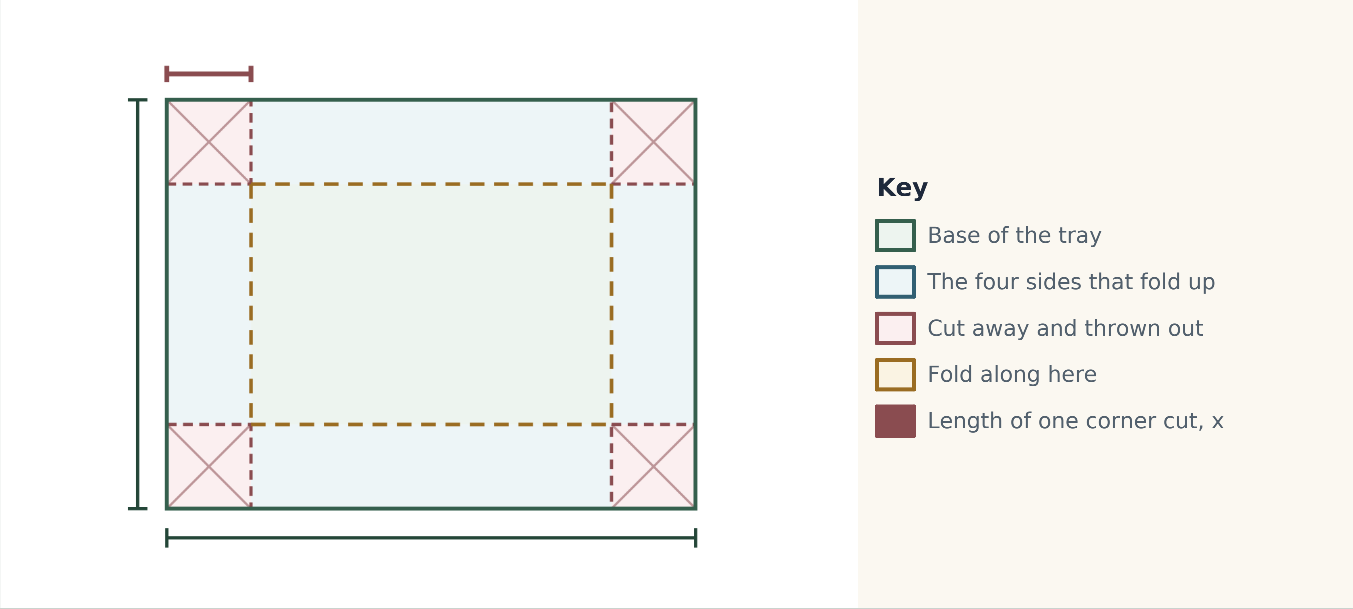
Cut four equal squares out of the corners of one 8.5 in by 11 in sheet, fold the flaps up, and tape the corners. That gives you an open-top tray.

Find the corner cut that makes the tray hold as much potting mix as possible **and still meets every rule below**.

Then convince somebody your answer is right. A number on its own is not an answer here.

The rules the tray has to follow

	Rule	Why it is there
R1	One sheet, 8.5 in by 11 in. All four corner squares the same size.	You get one sheet. No taping extra pieces on.
R2	Walls at least 1 in tall.	Shorter walls and the mix spills when someone slides the tray.
R3	The tray has to slide into the grow shelf. The opening is **5 in wide and 8 in deep**.	The shelf is already built. The tray has to fit it, not the other way round.
R4	Measure cuts to the nearest 1/8 in.	That is as fine as a ruler and a pair of scissors get you.



The flat sheet with the four corner squares taken out. The dashed lines are folds, not cuts.

What you hand in

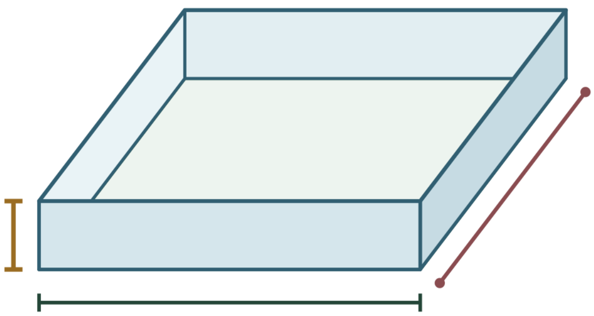
Parts A through G of this packet, filled in, plus the tray you actually built. Part G is the one that carries the most weight, so leave yourself time for it.

Before you start

You need: card stock or a manila folder, a ruler with eighths, scissors, tape, a pencil. A calculator helps in Part C. Graphing technology is optional everything here can be done by hand.

Part A · Build the model

Call the side of each corner square **x** inches. Everything else on the tray depends on that one number.



Key

- Base of the tray
- The four side walls
- Wall height, x
- Longer base side
- Shorter base side

The same sheet, folded. Nothing has been added only folded.

1. Fill in the three dimensions of the finished tray in terms of x.

Dimension	In terms of x
Height of the walls	
Longer side of the base	
Shorter side of the base	

2. Write the volume as a product of three factors. Do not expand it yet.

3. Now expand your answer to question 2 into standard form.

4. What is the degree of V(x)? What is the leading coefficient? Without graphing anything, what does the leading coefficient tell you about the two ends of the curve?

5. Set your factored form equal to zero and list all three zeros. Two of them are cut sizes you could physically make. One is not. Say which is which and why.

Part B · Say what x is allowed to be

A cubic will happily take $x = 20$ or $x = -3$. A sheet of card will not. This part is where you draw the line between the two.

6. What goes wrong if $x = 0$? Describe the object you would be holding.

7. The shorter side of the sheet is 8.5 in. What is the largest x can get before the tray stops existing? Show the inequality you solved.

8. Write the domain of V that comes from the sheet alone rules R2, R3 and R4 do not count yet. Use inequality notation.

Careful here

The domain of the *function* $4x^3 - 39x^2 + 93.5x$ is every real number. The domain of the *model* is much smaller. Those are two different questions and you are answering the second one.

Now bring in the design rules

9. Rule R2 says the walls are at least 1 in tall. Turn that into an inequality in x.

10. Rule R3 says the tray has to fit an opening 5 in wide and 8 in deep. The base measures $(11 - 2x)$ by $(8.5 - 2x)$. Write one inequality for the width fit and one for the depth fit, then solve both.

11. Put questions 8, 9 and 10 together. Write the one interval of x values that satisfies everything at once. This is your **feasible interval**.

Part C · Test some cut sizes

Before you graph anything, get a feel for the numbers. Fill the table in. Round volumes to two decimal places.

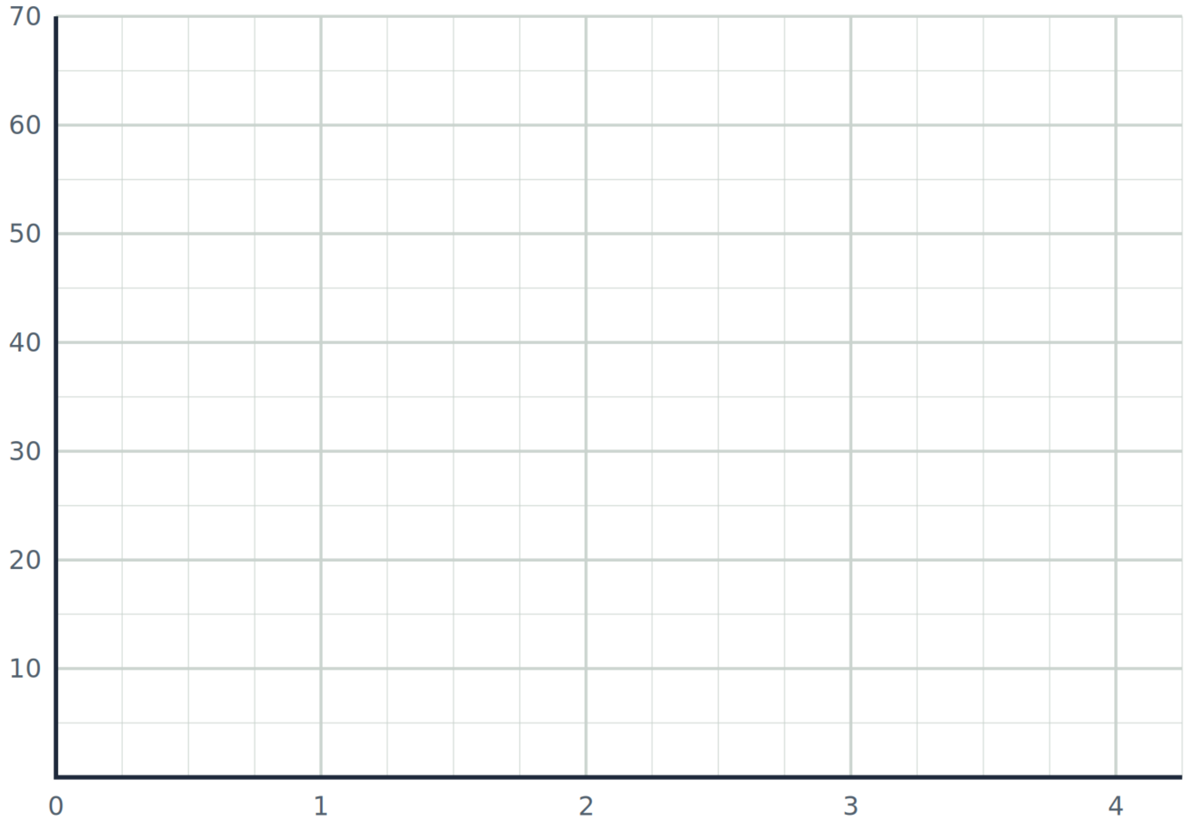
x (in)	Height	Longer base side	Shorter base side	Volume (in³)	Passes R2 and R3?
0.25					
0.50					
0.75					
1.00					
1.25					
1.50					
1.75					
2.00					
2.25					
2.50					
3.00					
3.50					
4.00					

12. Between which two of your x values does the volume stop climbing and start dropping?

13. Two different cut sizes in your table give volumes within a cubic inch of each other, but the trays look nothing alike. Find a pair and describe how the two trays differ.

Part D · Graph it

Plot every point from your Part C table, then join them with a smooth curve. Stop the curve at the ends of the domain you found in question 8. Do not run it past them.



Horizontal axis: corner cut x, in inches. Vertical axis: volume, in cubic inches.

14. Shade the part of the horizontal axis that rules R2 and R3 throw out. Then mark the feasible interval from question 11 on the same axis.

15. Mark the highest point on the curve and label it P. Mark the highest point that is inside the feasible interval and label it Q. Are P and Q the same point?

Part E · Pin down the maximum

Your graph gives you a rough answer. Now sharpen it with a table. Pick a starting range from your graph, then narrow it twice.

Round 1 step of 0.10

x						
V(x)						

Round 2 step of 0.02

x						
V(x)						

16. From your two rounds, the volume peaks near x = ______ in, giving V ≈ ______ in^3. State how confident you are in each decimal place and why.

17. Rule R4 says you can only cut to the nearest 1/8 in. List the eighths on either side of your answer to question 16 and work out the volume at each one.

18. Does the peak of the curve sit inside your feasible interval, or outside it? Answer in one sentence, then say what that means for your design.

Part F · Check the design against every rule

Write your chosen cut size at the top, then run it through all four rules. If it fails one, go back and choose again.

My chosen corner cut is x = _______________ in

Rule	The number I have to check	My value	Pass?
R1	All four corner squares equal, one sheet		
R2	Wall height, needs to be at least 1 in		
R3	Base width, needs to be at most 5 in		
R3	Base depth, needs to be at most 8 in		
R4	Cut written as a whole number of eighths		

19. What volume does your tray hold?

20. How much volume did the shelf rule cost you, compared with the very best cut on the curve? Give the loss in cubic inches and as a percentage.

Part G · Defend the design

This is the part that gets read most carefully. Write in full sentences. Point at your own numbers when you make a claim.

21. State your design in one sentence: the cut, the finished dimensions, and the volume.

22. Explain why no other cut size that follows all four rules beats yours. Your explanation has to cover every allowed x, not just the ones you happened to test.

23. Somebody proposes a cut that gives a bigger volume than yours. Give the number they most likely picked, and explain exactly which rule it breaks.

24. Suppose the grow shelf were rebuilt 6 in wide instead of 5 in. Would your design change? Say what the new answer would be and why.

Build it and see

The model says one thing. The card stock gets a vote too.

25. Cut and fold your tray. Measure the finished inside dimensions with a ruler and multiply them out.

	Predicted	Measured
Wall height		
Longer base side		
Shorter base side		
Volume		

26. Your measured volume is probably smaller than your predicted volume. Give two physical reasons why, and say roughly how many cubic inches each one costs you.

27. Fill the tray with rice or beans, pour it into a measuring cup, and record the volume in cups. One cup is about 14.4 in^3. How close did you get?

One last thing

Hold on to the tray. If your class does the 16 in by 10 in poster board version or the 36 in by 24 in cardboard version later, comparing the three trays side by side is where the pattern in the answers shows up.