How to use this

This is one worked response, written the way a student would write it. It is not the only right answer, and the wording is deliberately plain rather than polished. Hand it out after the project, not before.

Part A · Build the model

Each corner square has side x. When I fold the flaps up, the flap height is the side of the square I cut, so the walls are x inches tall.
The long side of the sheet is 11 in. I cut a square of side x off *each* end of it, so the base loses 2x and the longer base side is $11 - 2x$. The same thing happens on the 8.5 in side, so the shorter base side is $8.5 - 2x$.

Question 2 and 3

$V(x) = x(11 - 2x)(8.5 - 2x)$

Expanding: $(11 - 2x)(8.5 - 2x) = 93.5 - 22x - 17x + 4x^2 = 4x^2 - 39x + 93.5$

So $V(x) = 4x^3 - 39x^2 + 93.5x$

Check at $x = 1$: factored gives $1 \cdot 9 \cdot 6.5 = 58.5$, expanded gives $4 - 39 + 93.5 = 58.5$. They agree.

Question 4. Degree 3, leading coefficient 4. The degree is odd and the leading coefficient is positive, so far to the left the curve goes down and far to the right it goes up. Neither of those ends is a real tray, so the end behaviour is not much use here except as a check that my cubic is the right shape in the middle.

Question 5. Setting the factored form to zero gives $x = 0$, $x = 4.25$ and $x = 5.5$. At $x = 0$ I have not cut anything and there is no tray. At $x = 4.25$ the 8.5 in side has closed up completely and the base has no width. $x = 5.5$ would mean cutting 11 in of material off an 8.5 in side, which is not something a sheet of card can do. So two of the zeros are real events and one is only a root of the polynomial.

Part B · What x is allowed to be

Question 6. At $x = 0$ I would be holding the flat sheet. No walls, no volume.

Question 7. The shorter side has to survive: $8.5 - 2x > 0$, so $x < 4.25$. I checked the long side too, $11 - 2x > 0$ gives $x < 5.5$, but 4.25 is smaller so that is the one that binds. The short side always runs out first.

Question 8. Domain from the sheet: $0 < x < 4.25$.

Turning the brief into inequalities

Rule	Inequality	Solves to
R2, walls at least 1 in	$x \geq 1$	$x \geq 1$
R3, base fits a 5 in opening	$8.5 - 2x \leq 5$	**$x \geq 1.75$**
R3, base fits an 8 in depth	$11 - 2x \leq 8$	$x \geq 1.5$
R4, cuts to the nearest 1/8	x is a whole number of eighths	—

Question 11. All three of the first rules have to hold at once, so I take the strictest lower bound, which is 1.75, and keep the upper bound from the sheet. **Feasible interval: $1.75 \leq x < 4.25$.** The wall rule and the depth rule are already satisfied by everything in that interval, so the shelf width is doing all the work.

Something I noticed

The rule that ends up mattering is not the one that sounds most important. The wall height rule looks like the serious constraint when you read the brief, and it turns out to change nothing.

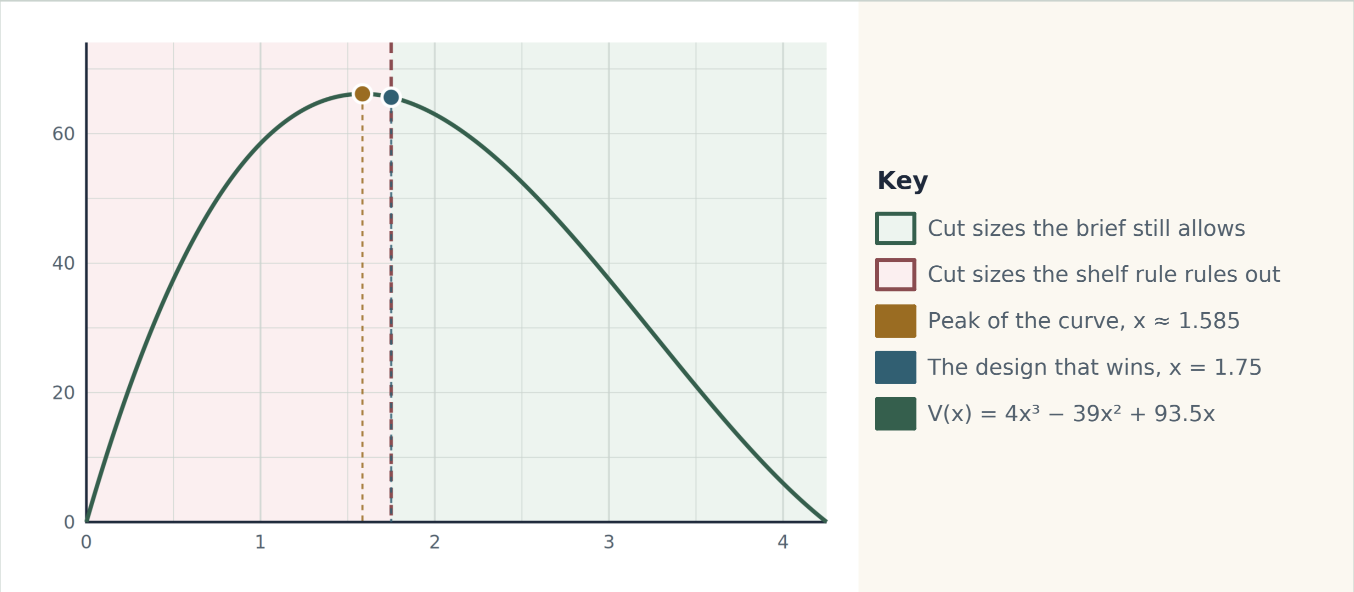
Part C · Testing cut sizes

x (in)	Base (in)	Volume (in^3)	Meets R2 and R3?
0.25	10.50 × 8.00	21.00	no
0.50	10.00 × 7.50	37.50	no
0.75	9.50 × 7.00	49.88	no
1.00	9.00 × 6.50	58.50	no
1.25	8.50 × 6.00	63.75	no
1.50	8.00 × 5.50	66.00	no
1.75	7.50 × 5.00	65.63	yes
2.00	7.00 × 4.50	63.00	yes
2.25	6.50 × 4.00	58.50	yes
2.50	6.00 × 3.50	52.50	yes
3.00	5.00 × 2.50	37.50	yes
3.50	4.00 × 1.50	21.00	yes
4.00	3.00 × 0.50	6.00	yes

Question 12. The volume climbs to 1.50 and has dropped again by 1.75, so the turn happens somewhere between them.

Question 13. $x = 1.00$ and $x = 2.25$ give exactly the same volume, 58.50 in^3. The first tray is 9 by 6.5 with 1 in walls, wide and shallow. The second is 6.5 by 4 with 2.25 in walls, narrow and deep. Same amount of soil, completely different tray. Once I saw that I understood why the graph has to come back down: a horizontal line can cross the curve twice.

Parts D and E · Graph and refine



My curve with the shelf rule marked. Everything to the left of the dashed line is a tray that will not go in the shelf.

Refining, step 0.10

x	1.30	1.40	1.50	1.60	1.70	1.80	1.90
V(x)	64.43	65.44	66.00	66.14	65.89	65.27	64.30

Refining, step 0.02

x	1.54	1.56	1.58	1.60	1.62	1.64
V(x)	66.107	66.135	66.148	66.144	66.125	66.089

Question 16. The peak is at about $x = 1.59$ in and the volume there is about 66.15 in³. I am confident about 1.5 and fairly confident about the second decimal, because the values at 1.58 and 1.60 differ by only 0.004. I would not claim a third decimal from this table.

Question 17. The eighths near the peak:

Cut	As a decimal	Volume (in³)	Base width (in)	Fits 5 in?
1 1/2	1.500	66.000	5.50	no
1 5/8	1.625	66.117	5.25	no
1 3/4	1.750	65.625	5.00	**yes**
1 7/8	1.875	64.570	4.75	yes

Question 18. The peak is outside the feasible interval. It sits at 1.585 and the shelf rule will not let me go below 1.75. So the design is not decided by the top of the curve at all; it is decided by the edge of what the shelf allows.

Parts F and G · The design and the defence

My design

Corner cut $x = 1\ 3/4$ in. Base 7.5 in by 5.0 in, walls 1.75 in tall, volume 65.625 in^3.

Rule	What I checked	My value	Pass
R1	Four equal squares, one sheet	1.75 in each	yes
R2	Walls at least 1 in	1.75 in	yes
R3	Base width at most 5 in	5.00 in	yes, exactly
R3	Base depth at most 8 in	7.50 in	yes
R4	A whole number of eighths	14 eighths	yes

Question 20. The best the curve can do anywhere is 66.148 in^3 at $x \approx 1.585$. My tray holds 65.625 in^3. The shelf rule costs me 0.523 in^3, which is $0.523 \div 66.148 \approx 0.79$ percent. Less than one percent, for a tray that actually goes in the shelf.

Question 22 · Why nothing allowed beats it

V has exactly one turning point on $0 < x < 4.25$, and it is a maximum, at $x \approx 1.585$. To the right of that point the curve only goes down. My feasible interval is $1.75 \le x < 4.25$, and every number in it is to the right of 1.585. So V is decreasing across the whole feasible interval, which means the largest allowed volume has to be at the smallest allowed x. That is 1.75. This covers every allowed cut, including the ones I never put in a table, because it is an argument about the shape of the curve rather than a list of tested values.

Question 23 · The answer somebody else would give

They would say 1 5/8 in, because it holds 66.117 in^3, which beats mine by half a cubic inch. It is the best eighth on the curve and it is the right answer to a different question. The base of that tray measures 7.75 in by 5.25 in, and the shelf opening is 5 in wide. It is a quarter of an inch too wide, so it breaks R3 and never gets into the shelf.

Question 24 · A 6 in shelf

With a 6 in opening the width rule becomes $8.5 - 2x \le 6$, so $x \ge 1.25$. The depth rule still gives $x \ge 1.5$, so the feasible interval becomes $1.5 \le x < 4.25$. Now the peak at 1.585 is inside it, so the constraint stops deciding the answer and the curve takes over. The nearest eighths are 1.5 and 1.625, and 1.625 wins with 66.117 in^3. So the design would change to a 1 5/8 in cut. One inch of shelf is worth about half a cubic inch of soil.

Building it

	Predicted	Measured
Wall height	1.75 in	1.69 in
Longer base side	7.50 in	7.44 in
Shorter base side	5.00 in	4.94 in
Volume	65.63 in^3	62.11 in^3

Question 26. My tray came out about 3.5 in^3 short, roughly five percent. Two reasons. First, the card has thickness, so the fold does not happen on a line, it happens over about a sixteenth of an inch, and every fold eats into the base. Four folds at a sixteenth each is most of the loss, maybe two and a half cubic inches. Second, the walls do not stand perfectly upright once they are taped; they lean in slightly at the top, which costs a bit more.

Question 27. The rice filled the tray to 4.3 cups. At 14.4 in^3 per cup that is about 61.9 in^3, which is within a quarter of a cubic inch of what I measured with the ruler. So the two physical measurements agree with each other, and both sit about five percent under the model. The model is not wrong; it just describes a tray made of something with no thickness.

What I would tell the garden club

Cut 1 3/4 in squares. The tray holds 65.6 in^3 on paper and about 62 in practice, and it slides into the shelf with nothing to spare. A 1 5/8 in cut holds more and does not fit, which makes it worth nothing at all.