The brief

The garden club needs seedling trays. You are going to design one, and you get one piece of poster board to do it with.

Your job

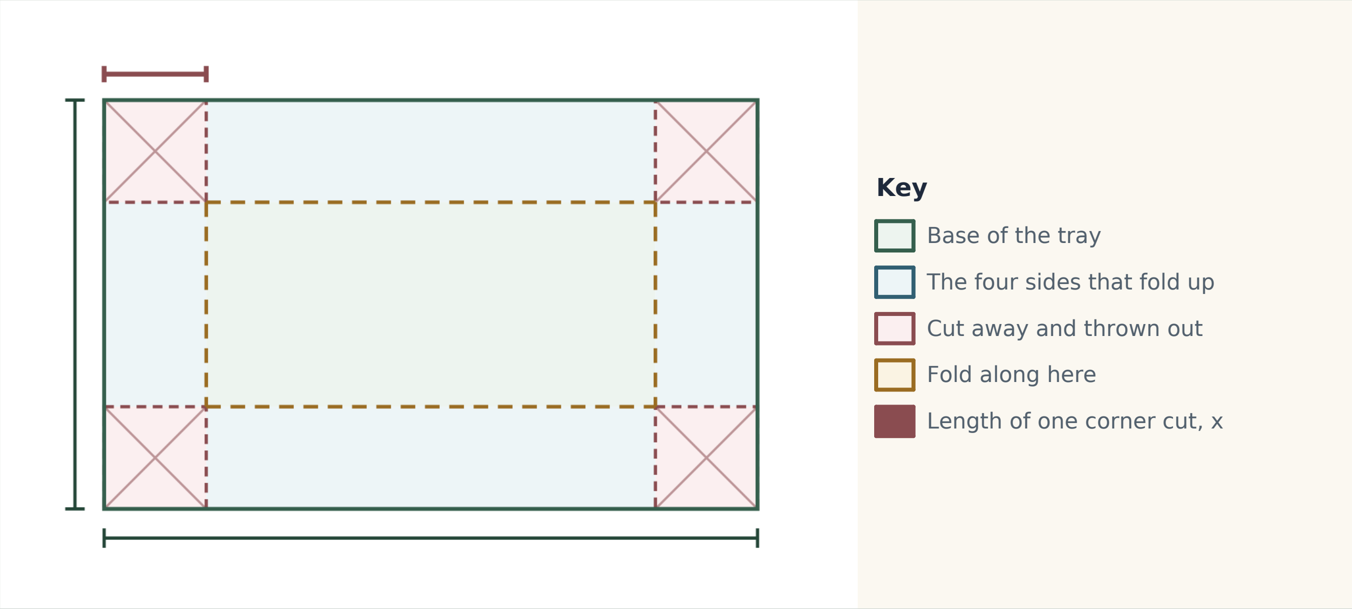
Cut four equal squares out of the corners of one 16 in by 10 in sheet, fold the flaps up, and tape the corners. That gives you an open-top tray.

Find the corner cut that makes the tray hold as much potting mix as possible **and still meets every rule below**.

Then convince somebody your answer is right. A number on its own is not an answer here.

The rules the tray has to follow

	Rule	Why it is there
R1	One sheet, 16 in by 10 in. All four corner squares the same size.	You get one sheet. No taping extra pieces on.
R2	Walls at least 1 in tall.	Shorter walls and the mix spills when someone slides the tray.
R3	The tray has to slide into the grow shelf. The opening is **5 in wide and 12 in deep**.	The shelf is already built. The tray has to fit it, not the other way round.
R4	Measure cuts to the nearest half inch.	That is as fine as a ruler and a pair of scissors get you on poster board.



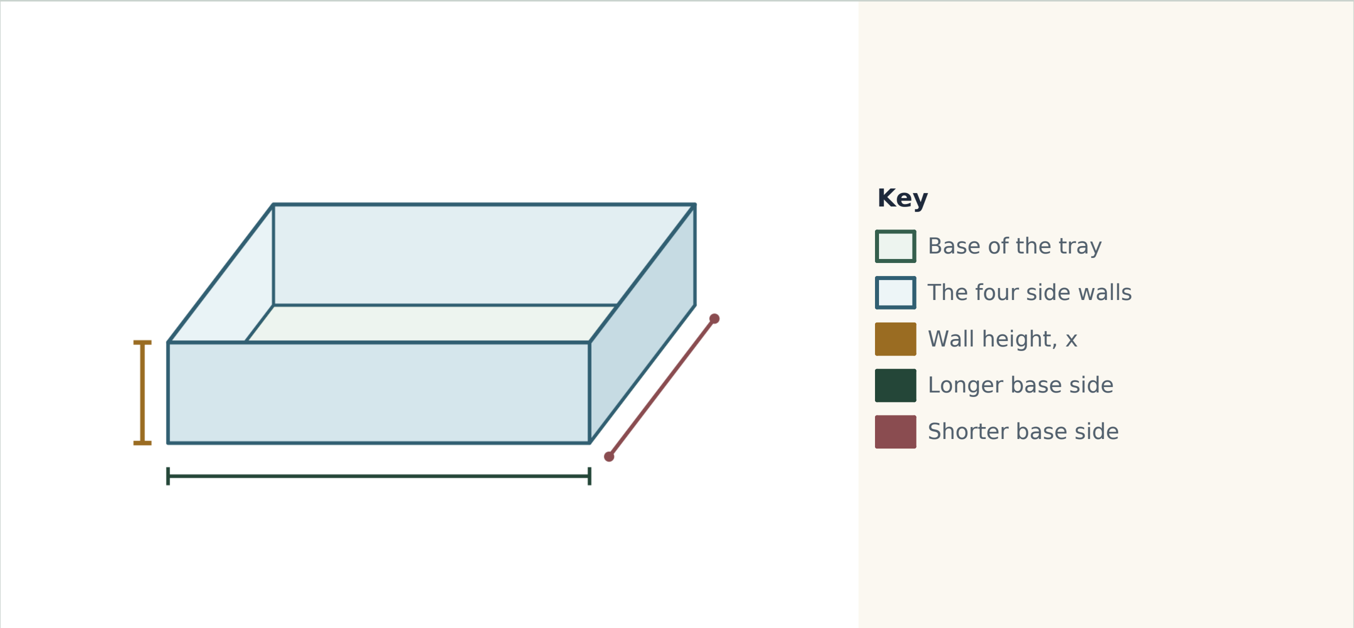
The flat sheet with the four corner squares taken out. The dashed lines are folds, not cuts.

Before you start

You need: poster board or a large sheet of card, a ruler, scissors, tape, a pencil. A calculator helps in Part C. You do not need graphing technology; every number in this packet can be worked out by hand.

Part A · Build the model

Call the side of each corner square **x** inches. Everything else about the tray depends on that one number.



The same sheet, folded up. Nothing has been added, only folded.

1. Fill in the three dimensions of the finished tray in terms of x.

Dimension	In terms of x
Height of the walls	
Longer side of the base	
Shorter side of the base	

Careful with this bit

A corner square is cut off **both** ends of every side. Trace one full edge of the drawing with your finger and count how many squares you cross before you write anything down.

2. Write the volume as a product of three factors. Do not expand it yet.

3. Now expand your answer to question 2 into standard form. Then check it: work out V(1) from the factored form and from the expanded form and make sure you get the same number both times.

4. Set your factored form equal to zero and list all three zeros. Two of them are cut sizes a real sheet could produce. One is not. Say which is which, and why.

Part B · Say what x is allowed to be

A cubic will happily take $x = 20$ or $x = -3$. A sheet of poster board will not. This part is where you draw the line between the two.

5. What goes wrong if $x = 0$? Describe the object you would be holding.

6. The shorter side of the sheet is 10 in. How large can x get before the tray stops existing? Show the inequality you solved. Check the 16 in side too, and say which of the two runs out first.

7. Write the domain of V that comes from the sheet alone. Rules R2, R3 and R4 do not count yet. Use inequality notation.

Two different questions

The domain of the *function* $4x^3 - 52x^2 + 160x$ is every real number. The domain of the *model* is much smaller. You are answering the second one.

Now bring in the design rules

8. Rule R2 says the walls are at least 1 in tall. Turn that into an inequality in x.

9. Rule R3 says the tray has to fit an opening 5 in wide and 12 in deep. The base measures $(16 - 2x)$ by $(10 - 2x)$. Write one inequality for the width fit and one for the depth fit, then solve both.

10. Put questions 7, 8 and 9 together. Write the one interval of x values that satisfies everything at once. This is your **feasible interval**.

Part C · Test every allowed cut size

Rule R4 says cuts go to the nearest half inch, so there are only nine cut sizes worth writing down. Fill the whole table in, including the ones that break a rule, because you will need them in Part D.

x (in)	Height	Longer base side	Shorter base side	Volume (in³)	Passes R2 and R3?
0.5					
1.0					
1.5					
2.0					
2.5					
3.0					
3.5					
4.0					
4.5					

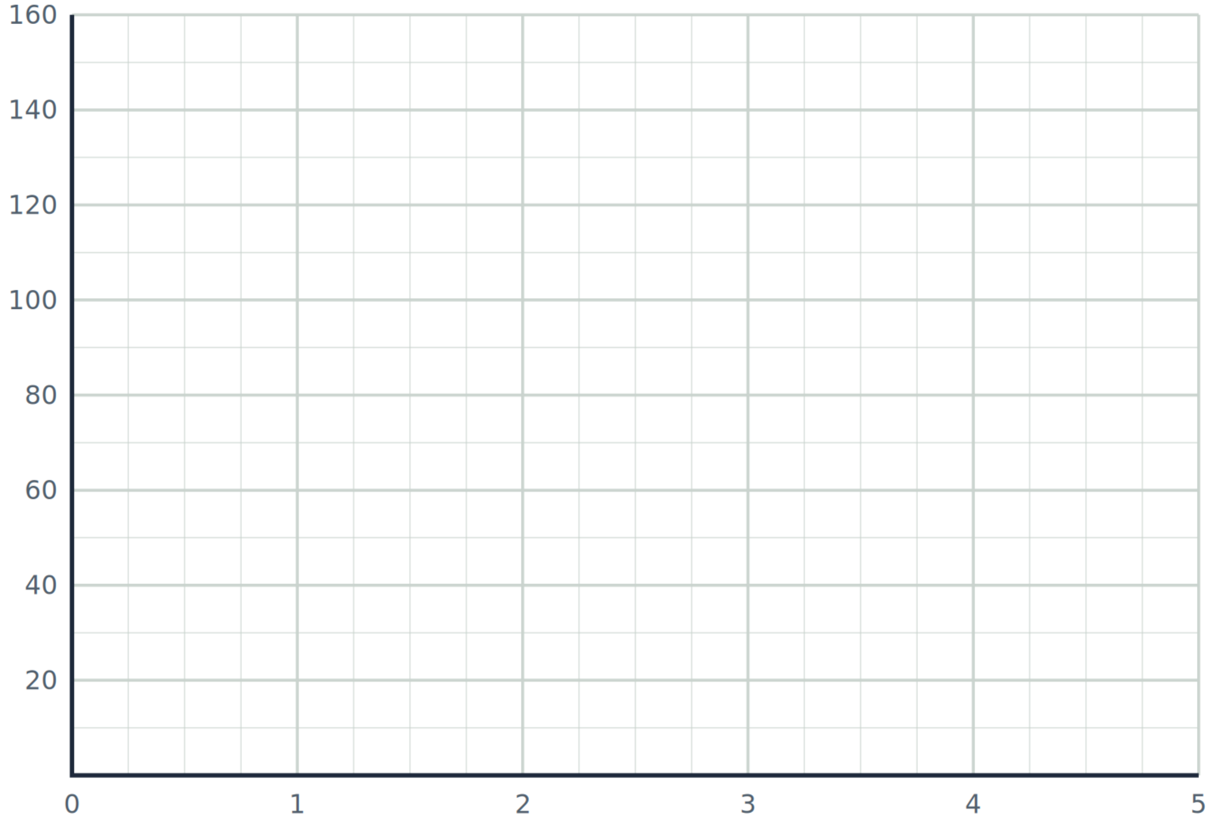
11. Which cut size gives the biggest volume of all nine?

12. Which cut size gives the biggest volume **out of the ones that pass every rule**? Are your answers to 11 and 12 the same?

13. Two different cut sizes in your table give volumes that are close to each other, but the two trays look nothing alike. Find a pair and describe how the trays differ.

Part D · Graph it

Plot all nine points from your Part C table, then join them with a smooth curve. Stop the curve at the ends of the domain you found in question 7. Do not run it past them.



Horizontal axis: corner cut x, in inches. Vertical axis: volume, in cubic inches.

14. Shade the part of the horizontal axis that the shelf rule throws out. Then mark your feasible interval from question 10 on the same axis.

15. Mark the highest point on the whole curve and label it P. Mark the highest point that is inside the shaded-in feasible interval and label it Q. Are P and Q the same point?

16. Look at your curve to the right of P. Is it going up, going down, or doing both? Answer in one sentence.

Part E · Check your design against every rule

Write your chosen cut size at the top, then run it through all four rules. If it fails one, go back and choose again.

My chosen corner cut is x = _______________ in

Rule	The number I have to check	My value	Pass?
R1	All four corner squares equal, one sheet		
R2	Wall height, needs to be at least 1 in		
R3	Base width, needs to be at most 5 in		
R3	Base depth, needs to be at most 12 in		
R4	Cut is a whole number of half inches		

17. What volume does your tray hold?

18. How much volume did the shelf rule cost you, compared with the best cut on the whole curve? Give the loss in cubic inches and as a percentage.

Part F · Defend the design

This is the part that gets read most carefully. Write in full sentences, and point at your own numbers when you make a claim.

19. State your design in one sentence: the cut, the finished dimensions, and the volume.

20. Somebody proposes a cut that gives a bigger volume than yours. Give the number they almost certainly picked, and explain exactly which rule it breaks and by how much.

21. Explain why no allowed cut beats yours. Your answer to question 16 is the key: if the curve only goes down after the peak, and every allowed cut is past the peak, what does that tell you about where the best allowed cut has to be?

22. Suppose the grow shelf were rebuilt 6 in wide instead of 5 in. Would your design change? Say what the new answer would be and why.

Build it and see

The model says one thing. The poster board gets a vote too.

23. Cut and fold your tray. Measure the finished inside dimensions with a ruler and multiply them out.

	Predicted	Measured
Wall height		
Longer base side		
Shorter base side		
Volume		

24. Your measured volume is probably smaller than your predicted volume. Give two physical reasons why.

25. Fill the tray with rice or dry beans, pour it into a measuring cup, and record the volume in cups. One cup is about 14.4 in^3. How close did you get?