Open-Top Box Design Challenge

Poster board edition · 16 in by 10 in sheet · Teacher answer key

Burke Tutoring in Fremont
Polynomial modeling project

What changed from the letter-sheet version

Same construction, same design brief, easier arithmetic. On a 16 in by 10 in sheet the maximum lands on a whole number, so the two rounds of estimation disappear. Rule R4 moves from eighths to half inches, which leaves exactly nine cut sizes to check the whole of Part C fits on one page.

What did **not** change: the shelf rule still pushes the peak out of the feasible interval, so students still have to notice that the best answer is not allowed, and still have to argue over an interval rather than off a table. Here the constraint costs 4.5 percent of the volume rather than 0.8 percent, which students find much easier to care about.

The model

$V(x) = x(16 - 2x)(10 - 2x) = 4x^3 - 52x^2 + 160x$

Zeros at $x = 0$, $x = 5$ and $x = 8$. Only the first two are reachable on a real sheet.

Domain from the sheet alone: $0 < x < 5$.

Peak of the curve: **$x = 2$ in, $V = 144$ in³**, and it is exact. $V'(x) = 12x^2 - 104x + 160 = 4(x - 2)(3x - 20)$, so the critical values are $x = 2$ and $x = 20/3 \approx 6.67$, and only $x = 2$ is in the domain.

Feasible interval once the brief is applied: **$2.5 \le x < 5$**.

The design: **x = 2.5 in, base 11 in by 5 in, walls 2.5 in, V = 137.5 in³**.

Part C, completed

x (in)	Height	Base (in)	Volume (in³)	R2?	R3 width	R3 depth	Passes?
0.5	0.5	15 × 9	67.5	no	no	no	no
1.0	1.0	14 × 8	112	yes	no	no	no
1.5	1.5	13 × 7	136.5	yes	no	no	no
2.0	2.0	12 × 6	**144**	yes	no	yes	no
2.5	2.5	11 × 5	**137.5**	yes	yes	yes	**yes**
3.0	3.0	10 × 4	120	yes	yes	yes	yes
3.5	3.5	9 × 3	94.5	yes	yes	yes	yes
4.0	4.0	8 × 2	64	yes	yes	yes	yes
4.5	4.5	7 × 1	31.5	yes	yes	yes	yes

Every volume in the table is a whole number or a half. No student needs a calculator for more than the multiplication.

Answers, question by question

Part A

Q	Answer
1	Height x, longer base side 16 − 2x, shorter base side 10 − 2x.
2	$V(x) = x(16 - 2x)(10 - 2x)$.
3	$V(x) = 4x^3 - 52x^2 + 160x$. Check at $x = 1$: factored gives $1 \cdot 14 \cdot 8 = 112$, expanded gives $4 - 52 + 160 = 112$.
4	x = 0, 5 and 8. At x = 0 nothing is cut and there is no tray. At x = 5 the 10 in side has closed to zero width. x = 8 would mean cutting 16 in off a 10 in side, which no sheet can do.

Part B

Q	Answer
5	A flat sheet. No walls, no volume.
6	$10 - 2x > 0$ gives $x < 5$. The 16 in side gives $x < 8$. The short side runs out first, so 5 is the one that binds.
7	$0 < x < 5$.
8	$x \geq 1$.
9	Width fit: $10 - 2x \leq 5$ gives $\mathbf{x \geq 2.5}$. Depth fit: $16 - 2x \leq 12$ gives $x \geq 2$. Both have to hold.
10	$2.5 \leq x < 5$. The shelf width is the binding rule; the wall rule and the depth fit are already satisfied by everything in that interval.

Parts C and D

Q	Answer
11	x = 2 in, with 144 in^3.
12	x = 2.5 in, with 137.5 in^3. No, they are not the same, and noticing that is the whole project.
13	The clean pair is x = 1.5 (136.5 in^3) and x = 2.5 (137.5 in^3), a difference of one cubic inch. The first tray is 13 by 7 with 1.5 in walls, wide and shallow; the second is 11 by 5 with 2.5 in walls, narrower and deeper. Almost the same amount of soil, very different tray. Some students offer 1.0 and 3.0 instead (112 and 120), which makes the same point less sharply.
15	P is the peak at (2, 144). Q is at (2.5, 137.5), at the left end of the feasible interval. They are different points.
16	It only goes down. Students who say “both” have usually run the curve past x = 5 and should be sent back to question 7.

Parts E and F

Q	Answer
Design	$x = 2.5$ in. Base 11 in by 5 in, walls 2.5 in, volume 137.5 in³. The base width is 5 in exactly, so the tray fits the opening with nothing to spare.
17	137.5 in³.
18	$144 - 137.5 = 6.5$ in³, and $6.5 \div 144 \approx 4.5$ percent.
20	They picked $x = 2$ in, which holds 144 in³. That tray has a base 12 in by 6 in. The shelf opening is 5 in wide, so it is a full inch too wide and fails R3. Worth noticing: its 12 in depth fits the shelf exactly, so a student who only checks the depth will think it passes.
21	The argument, in the form students should reach: the curve rises to a single peak at $x = 2$ and falls after it. Every allowed cut is 2.5 or more, so every allowed cut is past the peak, on the falling part. That means the largest allowed volume has to be at the smallest allowed cut, which is 2.5. This covers cut sizes nobody tested, because it is a statement about the shape of the curve rather than a list of values.
22	Yes, it changes. A 6 in opening gives $10 - 2x \le 6$, so $x \ge 2$, and the depth rule also gives $x \ge 2$. The feasible interval becomes $2 \le x < 5$, and the peak at $x = 2$ is now inside it. So the answer becomes $x = 2$ in with 144 in³, and the constraint stops deciding the design at all.

Part G

Measured volumes usually land 4 to 8 percent under the prediction. The two reasons students should reach: poster board has thickness, so each fold happens over a strip rather than on a line and eats into the base; and taped walls lean inward slightly at the top. A tray that measures 10.9 by 4.9 by 2.45 holds about 130.9 in³, which is a typical result.

Where students get stuck

Subtracting one x instead of two. $V = x(16 - x)(10 - x)$ is the most common wrong model. Send them back to the drawing to trace an edge.

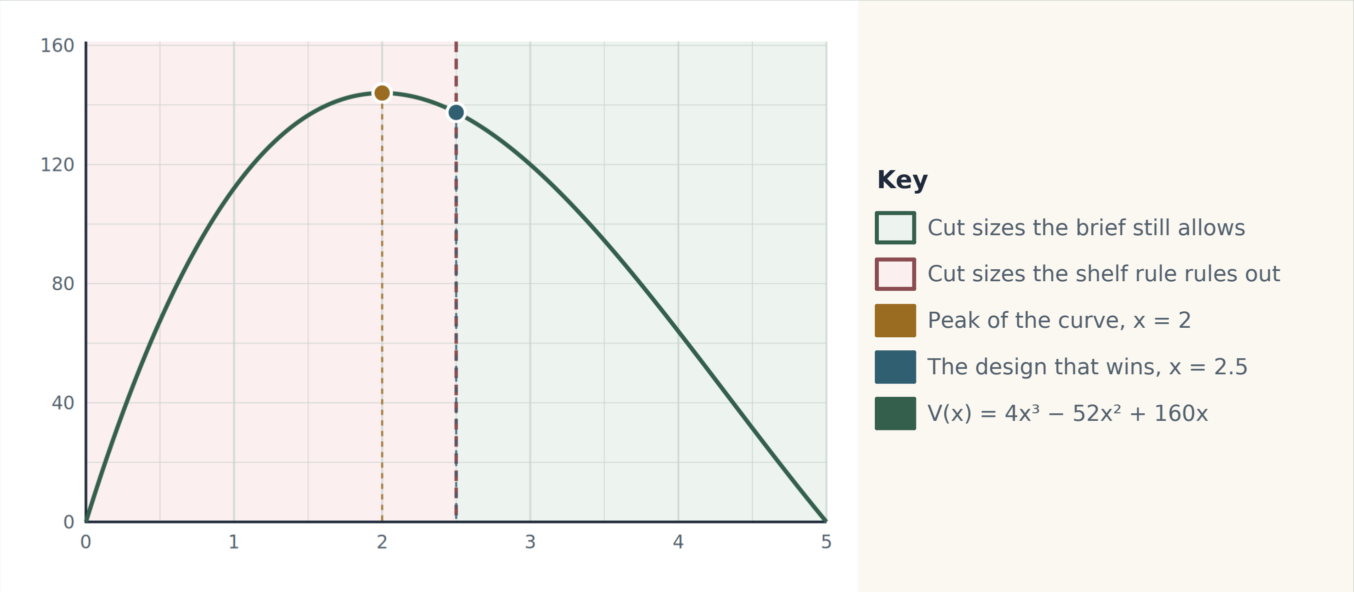
Using the long side for the domain. Some write $0 < x < 8$. Ask what the tray looks like at $x = 6$ and let them try to draw it.

Stopping at the peak. This is the failure the brief is built to catch. Do not warn them in advance. Let Part E catch it and let them fix it.

Checking the depth and forgetting the width. At $x = 2$ the depth fits exactly, so this error looks like a pass. Ask for both inequalities every time.

Arguing from the table alone. “I checked all nine” is true here, which makes question 21 harder to motivate than it looks. Push them to answer it without the table: what if cuts could be any size at all, not just halves?

The graph, for projecting



The gap between the gold point and the blue point is what the shelf rule costs. On this sheet it is 6.5 in^3, which is visible from the back of the room.

Running it

When	What
First 10 min	Hand each pair a sheet and let them cut a tray with any corner size they like. No mathematics yet. Stand three visibly different trays at the front.
Next 25 min	Parts A and B. Stop the class when the first pair writes $16 - x$ and put both versions on the board without saying which is right.
Next 20 min	Parts C and D. The table is short enough to finish in class, which is the main reason this version fits in less time than the letter-sheet one.
Last 20 min	Parts E and F. The moment a pair works out that a 2 in cut does not fit the shelf is the moment the project starts working. Part F in silence, individually.
Homework	Part G, the build and measure, if you did not get to it.

If a pair finishes early

Ask them for **every** cut that gives exactly 112 in^3. They already know $x = 1$ works from their table, so $(x - 1)$ is a factor; dividing leaves $x^2 - 12x + 28$, giving $x = 6 \pm 2\sqrt{2}$, that is 3.172 and 8.828. Three roots, two of which a real sheet can produce. Same question with 120 in^3 gives $x = 3$ and $x = 5 \pm \sqrt{15}$, that is 1.127 and 8.873.

The rubric works with this version unchanged, since its criteria are written to fit any of the three sheet sizes. The extension questions on this site now use the same 16 in by 10 in sheet as this packet, so the numbers line up directly.