The Open-Top Box

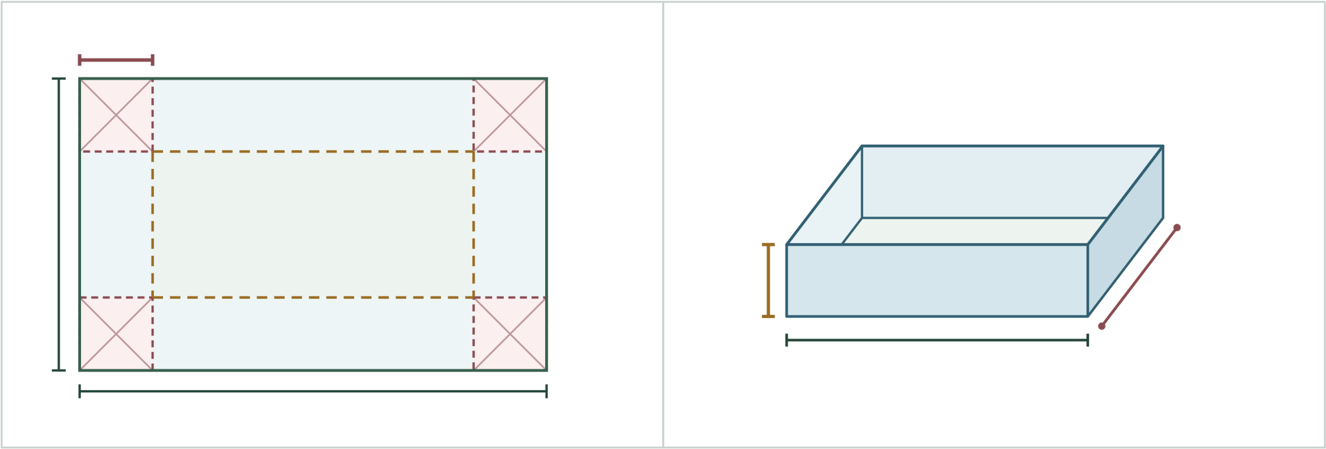
Express edition · one class period · Name ____________________

Burke Tutoring in Fremont
Polynomial modeling project

What you are doing

Cut four equal squares out of the corners of a 16 in by 10 in sheet. Fold the sides up and tape the corners. You get a tray with no lid.

Find the corner cut that makes the tray hold the most and still fits on the shelf. The shelf opening is 5 in wide, so the short side of your tray cannot be more than 5 in.



Left: the flat sheet, corner squares removed, dashed lines are folds. Right: the same sheet folded up. Rose is thrown away, green is the base, blue folds up into the walls.

Step 1 · The three dimensions

Each corner square has side **x** inches. One is done for you.

	Dimension of the tray	In terms of x
a	How tall the walls are	**x**
b	The longer side of the base	16 − ______
c	The shorter side of the base	10 − ______

Hint

You cut a square off **each end** of every side, not just one end. Put your finger on the left edge of the flat drawing and slide it right, counting the squares you cross.

Step 2 · The volume formula

Volume of a box is length × width × height. Fill in the three brackets using your answers above.

V = (_____________) × (_____________) × (_____________)

Step 3 · Try five cut sizes

The first row is done for you. Fill in the rest.

Cut x (in)	Longer base side	Shorter base side	Height	Volume (in^3)
1	14	8	1	14 × 8 × 1 = 112
1.5				
2				
2.5				
3				

Step 4 · The biggest tray

1. Circle the row with the **biggest volume**. Which cut is it, and how much does that tray hold?

Step 5 · Now check the shelf

The shelf opening is 5 in wide. Go back to your table and **cross out every row** where the shorter base side is bigger than 5 in, because those trays are too wide to go in.

2. Did the row you circled in Step 4 get crossed out?

3. Out of the rows that are **left**, which cut gives the biggest volume? This is your design.

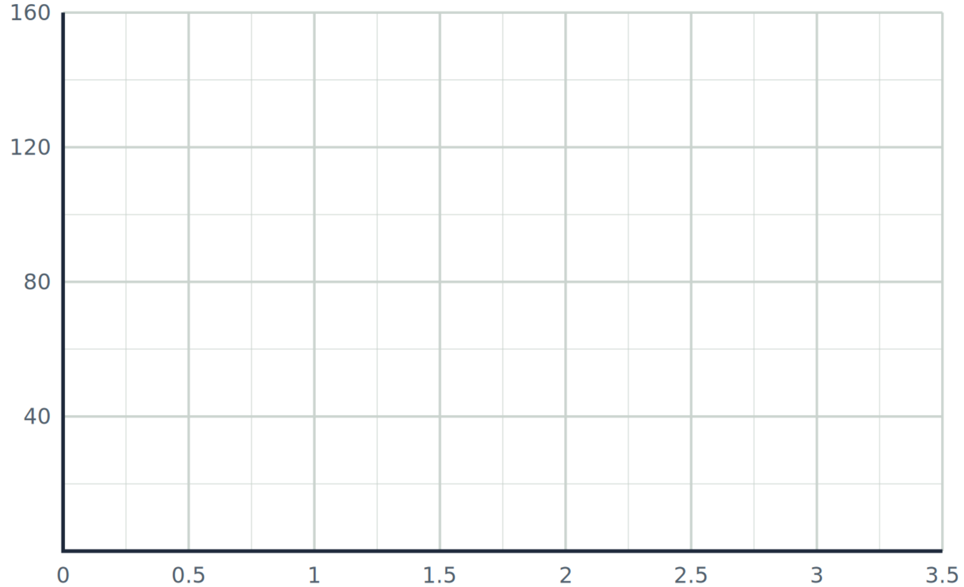
Step 6 · Explain your design

4. Somebody says "just cut 2 in squares, that holds the most." They are right that it holds the most. Explain in one or two sentences why it is still the wrong answer here.

5. How many cubic inches did the shelf rule cost you? (Biggest volume − your volume.)

Draw the picture

Plot your five points from the table, then join them with a smooth curve.



Across: the cut size x, in inches. Up: the volume, in cubic inches.

6. After the highest point, does your curve go up or down?

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7. You only tested five cuts. Use your answer to question 6 to explain why no **other** allowed cut, such as 4 in or 3.5 in or anything else past your design, could beat it.

Bonus, if you finish early

1. Multiply your three brackets out and write V as $4x^3 -$ ______ $x^2 +$ ______ x. Check it by putting $x = 1$ in and seeing if you get 112.

2. What is the biggest square you could possibly cut out of this sheet before there is no tray left at all? Explain what the sheet looks like at that point.

3. The shelf gets rebuilt 6 in wide instead of 5 in. Does your design change? To what?