The Open-Top Box

Express edition · one class period · Teacher answer key

What this version is

A one-period version of the open-top box task, stripped to the single idea worth taking away: **the biggest answer is not always the allowed one.**

No inequality notation, no domain written formally, no estimation, no percentages. Students check the shelf rule by crossing rows out of a table rather than by solving $10 - 2x \leq 5$. The longer packets remain available if you want the formal version later.

Runs in 25 to 35 minutes. Add 10 minutes if students build the tray.

Steps 1 and 2

Walls are **x** tall. Longer base side **16 − 2x**. Shorter base side **10 − 2x**. So $V = (16 - 2x)(10 - 2x)(x)$.

The near-universal error is writing $16 - x$ and $10 - x$. It is worth catching out loud: put both versions on the board, work out the base at $x = 2$ each way, and hold the folded tray up against a ruler.

Step 3 · The completed table

Cut x (in)	Longer base	Shorter base	Height	Volume (in^3)	Fits 5 in shelf?
1	14	8	1	112	no, cross out
1.5	13	7	1.5	136.5	no, cross out
2	12	6	2	**144**	no, cross out
2.5	11	5	2.5	**137.5**	**yes**
3	10	4	3	120	yes

Every number is a whole number or a half. No calculator needed beyond the multiplication.

Questions 1 to 5

Q	Answer
1	$x = 2$ in, holding 144 in^3.
2	Yes. Its shorter base side is 6 in, and the shelf is 5 in wide.
3	**$x = 2.5$ in, holding 137.5 in^3.** Base 11 in by 5 in, walls 2.5 in. It fits the shelf with nothing to spare.
4	Looking for: a 2 in cut makes a tray 6 in wide, the shelf opening is only 5 in, so the tray will not go in. A tray that holds more but does not fit is worth nothing. Accept any wording that names both the number and the rule.
5	$144 - 137.5 = 6.5$ in^3.

Questions 6 and 7 · the one that matters

6. Down. The curve rises to a single high point at $x = 2$ and falls after it.

7. Looking for: every allowed cut is 2.5 or bigger, so every allowed cut is past the top of the curve, on the part that is going down. Going further right only makes the tray smaller. So the best allowed cut is the smallest allowed cut, which is 2.5 in, and that covers cuts nobody tested.

This is the only genuinely abstract moment in the exercise. If a class is struggling, ask them to predict V at $x = 4$ from the shape of their curve before they compute it (it is 64), then ask whether they needed the arithmetic to know it would be small.

Bonus answers

	Answer
1	$V = 4x^3 - 52x^2 + 160x$. At $x = 1$: $4 - 52 + 160 = 112$. Matches the table.
2	Just under 5 in. At exactly 5 in the two cuts on the 10 in side meet in the middle, the base has no width left, and the sheet is four strips with nothing between them. Students often say 8 in, using the long side. Ask them to draw the tray at $x = 6$.
3	Yes. A 6 in opening lets the 2 in cut through, because that tray is 6 in wide. The design becomes $x = 2$ in with 144 in^3, and the shelf stops mattering at all.

Running it in 30 minutes

0 to 5 min. Hold up two trays you cut earlier with visibly different corner sizes. Ask which holds more. Take a vote. Do not settle it.

5 to 15 min. Steps 1 to 3, in pairs. Circulate for the $16 - x$ error.

15 to 22 min. Steps 4 and 5. Let the disappointment land when the circled row gets crossed out. That reaction is the point of the exercise.

22 to 32 min. Questions 4 to 7, individually and in silence. Question 7 is the one to discuss as a class before the bell.

Where to go next

The poster board packet uses the same sheet and the same shelf, but adds inequality notation, the full nine-row table, the formal domain and a written defence. It is the natural follow-on a day or two later, and students who did this exercise already know the answer, which frees them to concentrate on the reasoning.