Graphing Polynomial Functions — Part A: End Behavior

State the end behavior of each function: the direction of the left and right ends.

Name: ____________ Date: ____________ Period: ____________

Use the degree (even or odd) and the sign of the leading coefficient. For factored forms, multiply only the leading factors to find the leading term.

1. $f(x) = 2x^3 - 5x + 1$

   Left end ($x \to -\infty$): ________ Right end ($x \to +\infty$): ________

2. $f(x) = -x^4 + 3x^2 - 2$

   Left end ($x \to -\infty$): ________ Right end ($x \to +\infty$): ________

3. $f(x) = -4x^5 + x^2$

   Left end ($x \to -\infty$): ________ Right end ($x \to +\infty$): ________

4. $f(x) = 3x^6 - 7x^3 + 2$

   Left end ($x \to -\infty$): ________ Right end ($x \to +\infty$): ________

5. $f(x) = -2x^4 + 6x^3 - x + 9$

   Left end ($x \to -\infty$): ________ Right end ($x \to +\infty$): ________

6. $f(x) = -(x - 1)(x + 2)(x - 3)$

   Left end ($x \to -\infty$): ________ Right end ($x \to +\infty$): ________

Graphing Polynomial Functions — Part B: Zeros and Multiplicity

List every real zero, state its multiplicity, and say whether the graph crosses or touches the x-axis.

Name: ________ Date: ________ Period: ________

Odd multiplicity → the graph crosses. Even multiplicity → the graph touches and turns back.

1. $f(x) = (x - 4)(x + 1)$

   Zeros, multiplicity, and cross/touch:

2. $f(x) = (x - 2)^2(x + 3)$

   Zeros, multiplicity, and cross/touch:

3. $f(x) = x(x - 5)^3$

   Zeros, multiplicity, and cross/touch:

4. $f(x) = (x + 1)^2(x - 2)^2$

   Zeros, multiplicity, and cross/touch:

5. $f(x) = -2(x - 3)(x + 2)^2(x - 1)$

   Zeros, multiplicity, and cross/touch:

6. $f(x) = (x^2 - 9)(x + 4)$

Zeros, multiplicity, and cross/touch:

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Graphing Polynomial Functions — Part C: Sketching from Factored Form

State the degree, end behavior, zeros with multiplicity, and y-intercept, then sketch.

Name: ____________________ Date: ____________________ Period: ____________________

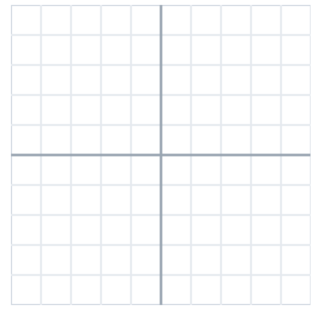
Set the ends from the leading term, mark each zero as a cross or a touch, plot the y-intercept, and draw a smooth curve. A degree-n graph has at most $n - 1$ turning points.

1. $f(x) = (x + 2)(x - 1)(x - 3)$

   Degree: __________ End behavior: ____________________

   Zeros & multiplicity: ______________________________

   y-intercept: ____________

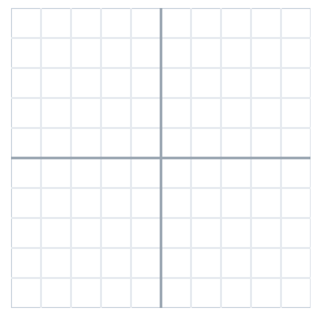


2. $f(x) = -(x - 1)^2(x + 2)$

   Degree: __________ End behavior: ____________________

   Zeros & multiplicity: ______________________________

   y-intercept: ____________

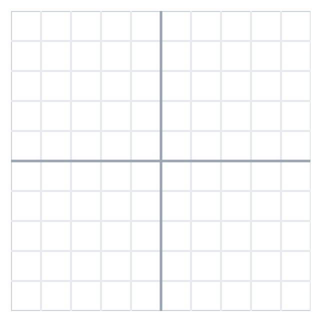


3. $f(x) = x^2(x - 3)$

   Degree: __________ End behavior: ____________________

   Zeros & multiplicity: ______________________________

   y-intercept: ____________

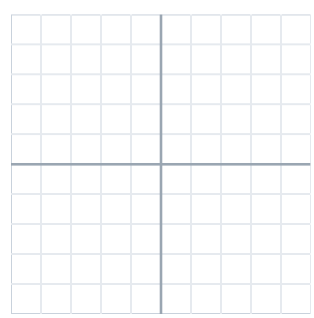


4. $f(x) = (x + 1)(x - 2)^2$

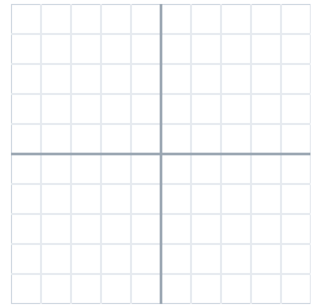
   Degree: __________ End behavior: ____________________

   Zeros & multiplicity: ______________________________

   y-intercept: ____________



5. $f(x) = -(x + 2)(x + 1)(x - 1)(x - 2)$

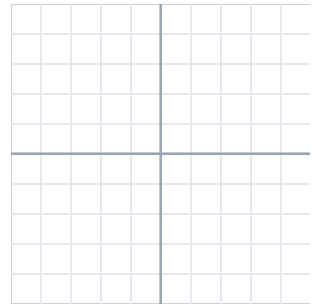


Degree: ________ End behavior: ________________

Zeros & multiplicity: ________________________

y-intercept: __________

6. $f(x) = (x - 1)^2(x + 2)^2$



Degree: ________ End behavior: ________________

Zeros & multiplicity: ________________________

y-intercept: __________

Graphing Polynomial Functions — Part D: Interpreting Graphs and Writing Equations

Answer each question using end behavior, zeros, and multiplicity.

Name: ____________ Date: ____________ Period: ____________

1. A polynomial has degree 5 and a negative leading coefficient. State its end behavior, the maximum number of x-intercepts, and the maximum number of turning points.

2. Write a possible equation for a degree-3 polynomial with a positive leading coefficient whose zeros are $x = -2$ (multiplicity 1) and $x = 1$ (multiplicity 2).

3. A graph crosses the x-axis at $x = -2$ and $x = 3$, touches the x-axis at $x = 1$, and both ends point up. Find the least possible degree and write one possible equation.

4. Match each function to its end behavior (both up, both down, down then up, or up then down): (a) $4x^4 - x^2$; (b) $-3x^3 + x$; (c) $2x^5 - 7$; (d) $-x^6 + 5x^2$.

5. A cubic polynomial has zeros at $x = -3$, $x = 0$, and $x = 2$, and its graph passes through the point $(1, -6)$. Find its equation.

6. Can a degree-4 polynomial have exactly three distinct real zeros? Explain, and give an example in factored form if it is possible.

Graphing Polynomial Functions — Answer Key

Complete worked answers for all 24 problems (Parts A–D).

Part A · End behavior

1. Degree 3, positive: left end down, right end up. As $x \to -\infty$, $f(x) \to -\infty$; as $x \to +\infty$, $f(x) \to +\infty$.
2. Degree 4, negative: both ends down. As $x \to \pm\infty$, $f(x) \to -\infty$.
3. Degree 5, negative: left end up, right end down.
4. Degree 6, positive: both ends up.
5. Degree 4, negative: both ends down.
6. Leading term $-x^3$: degree 3, negative. Left end up, right end down.

Part B · Zeros and multiplicity

1. $x = 4$ (mult. 1, crosses); $x = -1$ (mult. 1, crosses).
2. $x = 2$ (mult. 2, touches); $x = -3$ (mult. 1, crosses).
3. $x = 0$ (mult. 1, crosses); $x = 5$ (mult. 3, crosses with a flattening).
4. $x = -1$ (mult. 2, touches); $x = 2$ (mult. 2, touches).
5. $x = 3$ (mult. 1, crosses); $x = -2$ (mult. 2, touches); $x = 1$ (mult. 1, crosses).
6. Factors to $(x - 3)(x + 3)(x + 4)$: $x = 3, -3, -4$, each mult. 1, all crossing.

Part C · Sketching from factored form

1. Degree 3, positive; left down / right up. Zeros $-2, 1, 3$ (all cross). y-intercept $(0, 6)$.
2. Degree 3, leading term $-x^3$; left up / right down. Zeros $x = 1$ (touch), $x = -2$ (cross). y-intercept $(0, -2)$.
3. Degree 3, positive; left down / right up. Zeros $x = 0$ (touch), $x = 3$ (cross). y-intercept $(0, 0)$.
4. Degree 3, positive; left down / right up. Zeros $x = -1$ (cross), $x = 2$ (touch). y-intercept $(0, 4)$.
5. Degree 4, negative; both ends down. Zeros $-2, -1, 1, 2$ (all cross). y-intercept $(0, -4)$.
6. Degree 4, positive; both ends up. Zeros $x = 1$ (touch), $x = -2$ (touch). y-intercept $(0, 4)$.

Part D · Interpreting graphs and writing equations

1. Odd degree, negative leading coefficient: left end up, right end down. At most 5 x-intercepts and at most 4 turning points.

2. $f(x) = (x + 2)(x - 1)^2$ (any positive constant multiple is also correct).
3. Two crossings (odd) and one touch (even) give minimum degree 1 + 1 + 2 = 4; degree 4 with a positive leading coefficient makes both ends point up. One equation: $f(x) = (x + 2)(x - 3)(x - 1)^2$.
4. (a) both up; (b) up then down; (c) down then up; (d) both down.
5. $f(x) = a \cdot x(x + 3)(x - 2)$. Using $(1, -6)$: $-4a = -6$, so $a = 3/2$. Thus $f(x) = (3/2) \cdot x(x + 3)(x - 2)$.
6. Yes. If one zero has multiplicity 2 and two others have multiplicity 1, the degrees add to 4 with three distinct real zeros. Example: $f(x) = (x - 1)^2(x + 2)(x - 3)$.