Graphing Polynomial Functions — Answer Key

Complete worked answers for all 24 problems (Parts A–D).

Part A · End behavior

1. Degree 3, positive: left end down, right end up. As $x \to -\infty$, $f(x) \to -\infty$; as $x \to +\infty$, $f(x) \to +\infty$.
2. Degree 4, negative: both ends down. As $x \to \pm\infty$, $f(x) \to -\infty$.
3. Degree 5, negative: left end up, right end down.
4. Degree 6, positive: both ends up.
5. Degree 4, negative: both ends down.
6. Leading term $-x^3$: degree 3, negative. Left end up, right end down.

Part B · Zeros and multiplicity

1. $x = 4$ (mult. 1, crosses); $x = -1$ (mult. 1, crosses).
2. $x = 2$ (mult. 2, touches); $x = -3$ (mult. 1, crosses).
3. $x = 0$ (mult. 1, crosses); $x = 5$ (mult. 3, crosses with a flattening).
4. $x = -1$ (mult. 2, touches); $x = 2$ (mult. 2, touches).
5. $x = 3$ (mult. 1, crosses); $x = -2$ (mult. 2, touches); $x = 1$ (mult. 1, crosses).
6. Factors to $(x - 3)(x + 3)(x + 4)$: $x = 3, -3, -4$, each mult. 1, all crossing.

Part C · Sketching from factored form

1. Degree 3, positive; left down / right up. Zeros −2, 1, 3 (all cross). y-intercept (0, 6).
2. Degree 3, leading term $-x^3$; left up / right down. Zeros $x = 1$ (touch), $x = -2$ (cross). y-intercept (0, −2).
3. Degree 3, positive; left down / right up. Zeros $x = 0$ (touch), $x = 3$ (cross). y-intercept (0, 0).
4. Degree 3, positive; left down / right up. Zeros $x = -1$ (cross), $x = 2$ (touch). y-intercept (0, 4).
5. Degree 4, negative; both ends down. Zeros −2, −1, 1, 2 (all cross). y-intercept (0, −4).
6. Degree 4, positive; both ends up. Zeros $x = 1$ (touch), $x = -2$ (touch). y-intercept (0, 4).

Part D · Interpreting graphs and writing equations

1. Odd degree, negative leading coefficient: left end up, right end down. At most 5 x-intercepts and at most 4 turning points.

2. $f(x) = (x + 2)(x - 1)^2$ (any positive constant multiple is also correct).
3. Two crossings (odd) and one touch (even) give minimum degree $1 + 1 + 2 = 4$; degree 4 with a positive leading coefficient makes both ends point up. One equation: $f(x) = (x + 2)(x - 3)(x - 1)^2$.
4. (a) both up; (b) up then down; (c) down then up; (d) both down.
5. $f(x) = a \cdot x(x + 3)(x - 2)$. Using $(1, -6)$: $-4a = -6$, so $a = 3/2$. Thus $f(x) = (3/2) \cdot x(x + 3)(x - 2)$.
6. Yes. If one zero has multiplicity 2 and two others have multiplicity 1, the degrees add to 4 with three distinct real zeros. Example: $f(x) = (x - 1)^2(x + 2)(x - 3)$.
