

# Factoring Polynomials Reference Sheet

Patterns, method triggers, and quick checks - designed to sit beside the worksheet.

| Method                   | What to notice                                    | Pattern or steps                                                              | Example                                   |
|--------------------------|---------------------------------------------------|-------------------------------------------------------------------------------|-------------------------------------------|
| GCF                      | Always check first.                               | Divide every term by the largest shared number and variable power.            | $12x^3 - 18x^2 = 6x^2(2x - 3)$            |
| Difference of squares    | Two terms, subtraction, both perfect squares.     | $a^2 - b^2 = (a - b)(a + b)$                                                  | $9x^2 - 16 = (3x - 4)(3x + 4)$            |
| Perfect-square trinomial | First and last are squares; middle is $\pm 2ab$ . | $a^2 \pm 2ab + b^2 = (a \pm b)^2$                                             | $x^2 - 10x + 25 = (x - 5)^2$              |
| Trinomial, $a = 1$       | $x^2 + bx + c$                                    | Find numbers with product $c$ and sum $b$ .                                   | $x^2 + 7x + 12 = (x + 3)(x + 4)$          |
| AC method                | $ax^2 + bx + c$ with $a \neq 1$                   | Find product $ac$ and sum $b$ ; split $bx$ ; group.                           | $6x^2 + 11x + 3 = (3x + 1)(2x + 3)$       |
| Grouping                 | Usually four terms.                               | Pair terms, factor each pair, then factor the repeated binomial.              | $x^3 + 3x^2 - 4x - 12 = (x + 3)(x^2 - 4)$ |
| Cubes                    | Two perfect cubes.                                | $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$ ; $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$ | $x^3 - 27 = (x - 3)(x^2 + 3x + 9)$        |
| Quadratic form           | Only even powers such as $x^4$ and $x^2$ .        | Let $u = x^2$ , factor in $u$ , replace $u$ , factor again.                   | $x^4 - 13x^2 + 36 = (x^2 - 4)(x^2 - 9)$   |

## The AC Method in Five Moves

|   |                                                  |                                    |
|---|--------------------------------------------------|------------------------------------|
| 1 | Multiply $a \cdot c$ .                           | For $6x^2 + 11x + 3$ , $ac = 18$ . |
| 2 | Find two numbers with product $ac$ and sum $b$ . | 9 and 2.                           |
| 3 | Split the middle term.                           | $6x^2 + 9x + 2x + 3$ .             |
| 4 | Factor by grouping.                              | $3x(2x + 3) + 1(2x + 3)$ .         |
| 5 | Factor the repeated binomial.                    | $(3x + 1)(2x + 3)$ .               |

| Factoring completely checklist                                                                                                                                                                                                                                                          | Fast verification                                                                                                                                                                                                                                                                                                     |
|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <ol style="list-style-type: none"> <li>Put the polynomial in descending order.</li> <li>Factor out the GCF.</li> <li>Count terms and identify a pattern.</li> <li>Factor every new factor again.</li> <li>Stop only when each factor is prime over the chosen number system.</li> </ol> | <p>Multiply the factors back together. For trinomials, also check:</p> <ul style="list-style-type: none"> <li>- constant product</li> <li>- middle-term sum</li> <li>- leading-term product</li> </ul> <p>A factored answer is not finished if a difference of squares or a shared factor is still hiding inside.</p> |

Sign shortcut for  $x^2 + bx + c$ : If  $c$  is positive, the factor signs match and their sum has the sign of  $b$ . If  $c$  is negative, the signs differ; the larger absolute value controls the sign of  $b$ .