

Factoring Polynomials: Worked Answer Key

Each solution names the method, shows the key step, and gives the fully factored result.

Worksheet A: Factoring Foundations

1. $6x + 18$

Method: GCF

The greatest common factor is 6.

$$6x + 18 = 6(x + 3)$$

Final: $6(x + 3)$

2. $5x^2 - 20x$

Method: GCF

The greatest common factor is $5x$.

$$5x^2 - 20x = 5x(x - 4)$$

Final: $5x(x - 4)$

3. $x^2 + 7x + 12$

Method: Trinomial, $a = 1$

Find two numbers with product 12 and sum 7: 3 and 4.

$$x^2 + 7x + 12 = (x + 3)(x + 4)$$

Final: $(x + 3)(x + 4)$

4. $x^2 - x - 12$

Method: Trinomial, $a = 1$

Find two numbers with product -12 and sum -1: -4 and 3.

$$x^2 - x - 12 = (x - 4)(x + 3)$$

Final: $(x - 4)(x + 3)$

5. $x^2 - 25$

Method: Difference of squares

$$x^2 - 25 = x^2 - 5^2$$

$$a^2 - b^2 = (a - b)(a + b)$$

Final: $(x - 5)(x + 5)$

6. $4x^2 - 49$

Method: Difference of squares

$$4x^2 - 49 = (2x)^2 - 7^2$$

Use conjugates.

Final: $(2x - 7)(2x + 7)$

7. $x^2 + 10x + 25$

Method: Perfect-square trinomial

$$25 = 5^2 \text{ and } 10x = 2(x)(5).$$

$$x^2 + 10x + 25 = (x + 5)^2$$

Final: $(x + 5)^2$

8. $x^2 - 12x + 36$

Method: Perfect-square trinomial

$36 = 6^2$ and $-12x = 2(x)(-6)$.

$$x^2 - 12x + 36 = (x - 6)^2$$

Final: $(x - 6)^2$

9. $3x^2 + 12x + 12$

Method: GCF, then perfect square

Factor out 3: $3(x^2 + 4x + 4)$.

$$x^2 + 4x + 4 = (x + 2)^2$$

Final: $3(x + 2)^2$

10. $2x^2 + 10x + 12$

Method: GCF, then trinomial

Factor out 2: $2(x^2 + 5x + 6)$.

Product 6, sum 5: 2 and 3.

Final: $2(x + 2)(x + 3)$

11. $x^3 - 9x$

Method: GCF, then difference of squares

Factor out x: $x(x^2 - 9)$.

$$x^2 - 9 = (x - 3)(x + 3).$$

Final: $x(x - 3)(x + 3)$

12. $4y^2 + 20y + 24$

Method: GCF, then trinomial

Factor out 4: $4(y^2 + 5y + 6)$.

Product 6, sum 5: 2 and 3.

Final: $4(y + 2)(y + 3)$

Worksheet B: Mixed Factoring Methods

1. $6x^2 + 11x + 3$

Method: AC method

$ac = 18$. Use 9 and 2.

$$6x^2 + 9x + 2x + 3$$

$$3x(2x + 3) + 1(2x + 3)$$

Final: $(3x + 1)(2x + 3)$

2. $8x^2 - 2x - 3$

Method: AC method

$ac = -24$. Use -6 and 4.

$$8x^2 - 6x + 4x - 3$$

$$2x(4x - 3) + 1(4x - 3)$$

Final: $(2x + 1)(4x - 3)$

3. $12x^2 + 11x - 5$

Method: AC method

$ac = -60$. Use 15 and -4.

$$12x^2 + 15x - 4x - 5$$

$$3x(4x + 5) - 1(4x + 5)$$

Final: $(3x - 1)(4x + 5)$

4. $x^3 + 3x^2 - 4x - 12$

Method: Grouping, then difference of squares

$$x^2(x + 3) - 4(x + 3)$$

$$(x + 3)(x^2 - 4)$$

Factor $x^2 - 4$ again.

Final: $(x + 3)(x - 2)(x + 2)$

5. $2x^3 - 6x^2 + 5x - 15$

Method: Grouping

$$2x^2(x - 3) + 5(x - 3)$$

The repeated binomial is $x - 3$.

Final: $(x - 3)(2x^2 + 5)$

6. $x^3 - 27$

Method: Difference of cubes

$$x^3 - 27 = x^3 - 3^3$$

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

Final: $(x - 3)(x^2 + 3x + 9)$

7. $8x^3 + 125$

Method: Sum of cubes

$$8x^3 + 125 = (2x)^3 + 5^3$$

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

Final: $(2x + 5)(4x^2 - 10x + 25)$

8. $3x^4 - 15x^2$

Method: GCF

The greatest common factor is $3x^2$.

$x^2 - 5$ does not factor over the integers.

Final: $3x^2(x^2 - 5)$

9. $x^4 - 13x^2 + 36$

Method: Quadratic form

Let $u = x^2$: $u^2 - 13u + 36$.

$(u - 4)(u - 9)$

Replace u and factor both differences of squares.

Final: $(x - 2)(x + 2)(x - 3)(x + 3)$

10. $16x^4 - 81$

Method: Difference of squares, twice

$16x^4 - 81 = (4x^2)^2 - 9^2$.

$(4x^2 - 9)(4x^2 + 9)$

Factor $4x^2 - 9$ again.

Final: $(2x - 3)(2x + 3)(4x^2 + 9)$

11. $6a^2b - 24b$

Method: GCF, then difference of squares

Factor out $6b$: $6b(a^2 - 4)$.

$a^2 - 4 = (a - 2)(a + 2)$.

Final: $6b(a - 2)(a + 2)$

12. $4x^2 - 12xy + 9y^2$

Method: Perfect-square trinomial

$4x^2 = (2x)^2$ and $9y^2 = (3y)^2$.

$-12xy = 2(2x)(-3y)$.

Final: $(2x - 3y)^2$

13. $15m^2n + 10mn^2 - 5mn$

Method: GCF

The greatest common factor is $5mn$.

Divide each term by $5mn$.

Final: $5mn(3m + 2n - 1)$

14. $2x^3 + 7x^2 + 3x$

Method: GCF, then AC method

Factor out x : $x(2x^2 + 7x + 3)$.

$ac = 6$. Use 6 and 1.

$2x^2 + 6x + x + 3$

Final: $x(2x + 1)(x + 3)$

Worksheet C: Factoring Challenge

1. $12x^3 - 75x$

Method: GCF, then difference of squares

Factor out $3x$: $3x(4x^2 - 25)$.

$4x^2 - 25 = (2x - 5)(2x + 5)$.

Final: $3x(2x - 5)(2x + 5)$

2. $6x^3 + x^2 - 24x - 4$

Method: Grouping, then difference of squares

$x^2(6x + 1) - 4(6x + 1)$

$(6x + 1)(x^2 - 4)$

Factor $x^2 - 4$.

Final: $(6x + 1)(x - 2)(x + 2)$

3. $2x^4 - 10x^2 + 8$

Method: GCF, quadratic form, then squares

$2(x^4 - 5x^2 + 4)$

Let $u = x^2$: $2(u - 1)(u - 4)$.

Replace u and factor each difference of squares.

Final: $2(x - 1)(x + 1)(x - 2)(x + 2)$

4. $x^6 - 64$

Method: Difference of squares, then cubes

$x^6 - 64 = (x^3 - 8)(x^3 + 8)$.

Factor each cube expression.

Final: $(x - 2)(x^2 + 2x + 4)(x + 2)(x^2 - 2x + 4)$

5. $16x^4 - 40x^2y^2 + 25y^4$

Method: Perfect-square trinomial

$16x^4 = (4x^2)^2$ and $25y^4 = (5y^2)^2$.

$-40x^2y^2 = 2(4x^2)(-5y^2)$.

Final: $(4x^2 - 5y^2)^2$

6. $18x^2 - 33x - 30$

Method: GCF, then AC method

Factor out 3: $3(6x^2 - 11x - 10)$.

$ac = -60$. Use -15 and 4.

$6x^2 - 15x + 4x - 10$

Final: $3(3x + 2)(2x - 5)$

7. $4x^4 + 4x^2 - 15$

Method: Quadratic form

Let $u = x^2$: $4u^2 + 4u - 15$.

$(2u - 3)(2u + 5)$

Replace u with x^2 .

Final: $(2x^2 - 3)(2x^2 + 5)$

8. $x^4 + 2x^3 - 9x^2 - 18x$

Method: GCF, grouping, then difference of squares

Factor out x.

$$x[x^2(x + 2) - 9(x + 2)]$$

$$x(x + 2)(x^2 - 9)$$

Final: $x(x + 2)(x - 3)(x + 3)$

9. $16a^4 - 81b^4$

Method: Difference of squares, twice

$$(4a^2 - 9b^2)(4a^2 + 9b^2)$$

Factor $4a^2 - 9b^2$ again.

Final: $(2a - 3b)(2a + 3b)(4a^2 + 9b^2)$

10. $3x^5 - 21x^3 + 36x$

Method: GCF, quadratic form, then difference of squares

Factor out 3x: $3x(x^4 - 7x^2 + 12)$.

Let $u = x^2$: $(u - 3)(u - 4)$.

Replace u and factor $x^2 - 4$.

Final: $3x(x^2 - 3)(x - 2)(x + 2)$