How Do I Factor This Polynomial?

A simple guide

Start

1. Factor out the GCF first

$$6x^2 + 9x = 3x(2x + 3)$$

2. How many terms are left?

2 terms

Look for special patterns

- $a^2 - b^2 = (a - b)(a + b)$ — difference of squares
- $a^3 \pm b^3$ — sum/difference of cubes

3 terms

Does it look like $ax^2 + bx + c$?

- If $a = 1$, find two numbers
- If $a \neq 1$, use the AC method

Also check perfect square trinomials

4 terms

Try grouping

$$ax + ay + bx + by$$

Always check: Can it factor more?

Done!

Factoring Polynomials Reference Sheet

Patterns, method triggers, and quick checks - designed to sit beside the worksheet.

Method	What to notice	Pattern or steps	Example
GCF	Always check first.	Divide every term by the largest shared number and variable power.	$12x^3 - 18x^2 = 6x^2(2x - 3)$
Difference of squares	Two terms, subtraction, both perfect squares.	$a^2 - b^2 = (a - b)(a + b)$	$9x^2 - 16 = (3x - 4)(3x + 4)$
Perfect-square trinomial	First and last are squares; middle is $\pm 2ab$.	$a^2 \pm 2ab + b^2 = (a \pm b)^2$	$x^2 - 10x + 25 = (x - 5)^2$
Trinomial, $a = 1$	$x^2 + bx + c$	Find numbers with product c and sum b.	$x^2 + 7x + 12 = (x + 3)(x + 4)$
AC method	$ax^2 + bx + c$ with $a \neq 1$	Find product ac and sum b; split bx; group.	$6x^2 + 11x + 3 = (3x + 1)(2x + 3)$
Grouping	Usually four terms.	Pair terms, factor each pair, then factor the repeated binomial.	$x^3 + 3x^2 - 4x - 12 = (x + 3)(x^2 - 4)$
Cubes	Two perfect cubes.	$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$; $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$	$x^3 - 27 = (x - 3)(x^2 + 3x + 9)$
Quadratic form	Only even powers such as x^4 and x^2.	Let $u = x^2$, factor in u, replace u, factor again.	$x^4 - 13x^2 + 36 = (x^2 - 4)(x^2 - 9)$

The AC Method in Five Moves

1	Multiply $a \cdot c$.	For $6x^2 + 11x + 3$, $ac = 18$.
2	Find two numbers with product ac and sum b.	9 and 2.
3	Split the middle term.	$6x^2 + 9x + 2x + 3$.
4	Factor by grouping.	$3x(2x + 3) + 1(2x + 3)$.
5	Factor the repeated binomial.	$(3x + 1)(2x + 3)$.

Factoring completely checklist	Fast verification
1. Put the polynomial in descending order. 2. Factor out the GCF. 3. Count terms and identify a pattern. 4. Factor every new factor again. 5. Stop only when each factor is prime over the chosen number system.	Multiply the factors back together. For trinomials, also check: - constant product - middle-term sum - leading-term product A factored answer is not finished if a difference of squares or a shared factor is still hiding inside.

Sign shortcut for $x^2 + bx + c$: If c is positive, the factor signs match and their sum has the sign of b. If c is negative, the signs differ; the larger absolute value controls the sign of b.

Worksheet A: Factoring Foundations

GCF, basic trinomials, difference of squares, and perfect-square trinomials.

Name: ______________________________ Date: _________________

1. $6x + 18$ *Factor completely over the integers.*

2. $5x^2 - 20x$ *Factor completely over the integers.*

3. $x^2 + 7x + 12$ *Factor completely over the integers.*

4. $x^2 - x - 12$ *Factor completely over the integers.*

5. $x^2 - 25$ *Factor completely over the integers.*

6. $4x^2 - 49$ *Factor completely over the integers.*

Worksheet A: Factoring Foundations

Continue. Check every factor before you stop.

Name: ____________________________ Date: ________________

7. $x^2 + 10x + 25$ *Factor completely over the integers.*

8. $x^2 - 12x + 36$ *Factor completely over the integers.*

9. $3x^2 + 12x + 12$ *Factor completely over the integers.*

10. $2x^2 + 10x + 12$ *Factor completely over the integers.*

11. $x^3 - 9x$ *Factor completely over the integers.*

12. $4y^2 + 20y + 24$ *Factor completely over the integers.*

Worksheet B: Mixed Factoring Methods

Choose the method. Several problems require more than one pass.

Name: ______________________________ Date: _________________

1. $6x^2 + 11x + 3$ *Factor completely over the integers.*

2. $8x^2 - 2x - 3$ *Factor completely over the integers.*

3. $12x^2 + 11x - 5$ *Factor completely over the integers.*

4. $x^3 + 3x^2 - 4x - 12$ *Factor completely over the integers.*

5. $2x^3 - 6x^2 + 5x - 15$ *Factor completely over the integers.*

6. $x^3 - 27$ *Factor completely over the integers.*

7. $8x^3 + 125$ *Factor completely over the integers.*

Worksheet B: Mixed Factoring Methods

Continue. Factor every result again.

Name: ______________________________ Date: ________________

8. $3x^4 - 15x^2$

Factor completely over the integers.

9. $x^4 - 13x^2 + 36$

Factor completely over the integers.

10. $16x^4 - 81$

Factor completely over the integers.

11. $6a^2b - 24b$

Factor completely over the integers.

12. $4x^2 - 12xy + 9y^2$

Factor completely over the integers.

13. $15m^2n + 10mn^2 - 5mn$

Factor completely over the integers.

14. $2x^3 + 7x^2 + 3x$

Factor completely over the integers.

Worksheet C: Factoring Challenge

No method labels. Expect hidden GCFs, repeated patterns, and quadratic form.

Name: ______________________________ Date: ________________

1. $12x^3 - 75x$

Factor completely over the integers.

2. $6x^3 + x^2 - 24x - 4$

Factor completely over the integers.

3. $2x^4 - 10x^2 + 8$

Factor completely over the integers.

4. $x^6 - 64$

Factor completely over the integers.

5. $16x^4 - 40x^2y^2 + 25y^4$

Factor completely over the integers.

Worksheet C: Factoring Challenge

Continue. Verify by multiplying your factors back together.

Name: ______________________________ Date: ________________

6. $18x^2 - 33x - 30$

Factor completely over the integers.

7. $4x^4 + 4x^2 - 15$

Factor completely over the integers.

8. $x^4 + 2x^3 - 9x^2 - 18x$

Factor completely over the integers.

9. $16a^4 - 81b^4$

Factor completely over the integers.

10. $3x^5 - 21x^3 + 36x$

Factor completely over the integers.

Factoring Polynomials: Worked Answer Key

Each solution names the method, shows the key step, and gives the fully factored result.

Worksheet A: Factoring Foundations

1. $6x + 18$

Method: GCF
The greatest common factor is 6.
$6x + 18 = 6(x + 3)$
Final: $6(x + 3)$

2. $5x^2 - 20x$

Method: GCF
The greatest common factor is $5x$.
$5x^2 - 20x = 5x(x - 4)$
Final: $5x(x - 4)$

3. $x^2 + 7x + 12$

Method: Trinomial, $a = 1$
Find two numbers with product 12 and sum 7: 3 and 4.
$x^2 + 7x + 12 = (x + 3)(x + 4)$
Final: $(x + 3)(x + 4)$

4. $x^2 - x - 12$

Method: Trinomial, $a = 1$
Find two numbers with product -12 and sum -1: -4 and 3.
$x^2 - x - 12 = (x - 4)(x + 3)$
Final: $(x - 4)(x + 3)$

5. $x^2 - 25$

Method: Difference of squares
$x^2 - 25 = x^2 - 5^2$
$a^2 - b^2 = (a - b)(a + b)$
Final: $(x - 5)(x + 5)$

6. $4x^2 - 49$

Method: Difference of squares
$4x^2 - 49 = (2x)^2 - 7^2$
Use conjugates.
Final: $(2x - 7)(2x + 7)$

7. $x^2 + 10x + 25$

Method: Perfect-square trinomial
$25 = 5^2$ and $10x = 2(x)(5)$.
$x^2 + 10x + 25 = (x + 5)^2$
Final: $(x + 5)^2$

8. $x^2 - 12x + 36$

Method: Perfect-square trinomial
$36 = 6^2$ and $-12x = 2(x)(-6)$.
$x^2 - 12x + 36 = (x - 6)^2$
Final: $(x - 6)^2$

9. $3x^2 + 12x + 12$

Method: GCF, then perfect square
Factor out 3: $3(x^2 + 4x + 4)$.
$x^2 + 4x + 4 = (x + 2)^2$
Final: $3(x + 2)^2$

10. $2x^2 + 10x + 12$

Method: GCF, then trinomial
Factor out 2: $2(x^2 + 5x + 6)$.
Product 6, sum 5: 2 and 3.
Final: $2(x + 2)(x + 3)$

11. $x^3 - 9x$

Method: GCF, then difference of squares
Factor out x: $x(x^2 - 9)$.
$x^2 - 9 = (x - 3)(x + 3)$.
Final: $x(x - 3)(x + 3)$

12. $4y^2 + 20y + 24$

Method: GCF, then trinomial
Factor out 4: $4(y^2 + 5y + 6)$.
Product 6, sum 5: 2 and 3.
Final: $4(y + 2)(y + 3)$

Worksheet B: Mixed Factoring Methods

1. $6x^2 + 11x + 3$

Method: AC method
$ac = 18$. Use 9 and 2.
$6x^2 + 9x + 2x + 3$
$3x(2x + 3) + 1(2x + 3)$
Final: $(3x + 1)(2x + 3)$

2. $8x^2 - 2x - 3$

Method: AC method
$ac = -24$. Use -6 and 4.
$8x^2 - 6x + 4x - 3$
$2x(4x - 3) + 1(4x - 3)$
Final: $(2x + 1)(4x - 3)$

3. $12x^2 + 11x - 5$

Method: AC method
$ac = -60$. Use 15 and -4.
$12x^2 + 15x - 4x - 5$
$3x(4x + 5) - 1(4x + 5)$
Final: $(3x - 1)(4x + 5)$

4. $x^3 + 3x^2 - 4x - 12$

Method: Grouping, then difference of squares
$x^2(x + 3) - 4(x + 3)$
$(x + 3)(x^2 - 4)$
Factor $x^2 - 4$ again.
Final: $(x + 3)(x - 2)(x + 2)$

5. $2x^3 - 6x^2 + 5x - 15$

Method: Grouping
$2x^2(x - 3) + 5(x - 3)$
The repeated binomial is $x - 3$.
Final: $(x - 3)(2x^2 + 5)$

6. $x^3 - 27$

Method: Difference of cubes
$x^3 - 27 = x^3 - 3^3$
$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
Final: $(x - 3)(x^2 + 3x + 9)$

7. $8x^3 + 125$

Method: Sum of cubes
$8x^3 + 125 = (2x)^3 + 5^3$
$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
Final: $(2x + 5)(4x^2 - 10x + 25)$

8. $3x^4 - 15x^2$

Method: GCF
The greatest common factor is $3x^2$.
$x^2 - 5$ does not factor over the integers.
Final: $3x^2(x^2 - 5)$

9. $x^4 - 13x^2 + 36$

Method: Quadratic form
Let $u = x^2$: $u^2 - 13u + 36$.
$(u - 4)(u - 9)$
Replace u and factor both differences of squares.
Final: $(x - 2)(x + 2)(x - 3)(x + 3)$

10. $16x^4 - 81$

Method: Difference of squares, twice
$16x^4 - 81 = (4x^2)^2 - 9^2$.
$(4x^2 - 9)(4x^2 + 9)$
Factor $4x^2 - 9$ again.
Final: $(2x - 3)(2x + 3)(4x^2 + 9)$

11. $6a^2b - 24b$

Method: GCF, then difference of squares
Factor out 6b: $6b(a^2 - 4)$.
$a^2 - 4 = (a - 2)(a + 2)$.
Final: $6b(a - 2)(a + 2)$

12. $4x^2 - 12xy + 9y^2$

Method: Perfect-square trinomial
$4x^2 = (2x)^2$ and $9y^2 = (3y)^2$.
$-12xy = 2(2x)(-3y)$.
Final: $(2x - 3y)^2$

13. $15m^2n + 10mn^2 - 5mn$

Method: GCF
The greatest common factor is 5mn.
Divide each term by 5mn.
Final: $5mn(3m + 2n - 1)$

14. $2x^3 + 7x^2 + 3x$

Method: GCF, then AC method
Factor out x: $x(2x^2 + 7x + 3)$.
ac = 6. Use 6 and 1.
$2x^2 + 6x + x + 3$
Final: $x(2x + 1)(x + 3)$

Worksheet C: Factoring Challenge

1. $12x^3 - 75x$

Method: GCF, then difference of squares
Factor out $3x$: $3x(4x^2 - 25)$.
$4x^2 - 25 = (2x - 5)(2x + 5)$.
Final: $3x(2x - 5)(2x + 5)$

2. $6x^3 + x^2 - 24x - 4$

Method: Grouping, then difference of squares
$x^2(6x + 1) - 4(6x + 1)$
$(6x + 1)(x^2 - 4)$
Factor $x^2 - 4$.
Final: $(6x + 1)(x - 2)(x + 2)$

3. $2x^4 - 10x^2 + 8$

Method: GCF, quadratic form, then squares
$2(x^4 - 5x^2 + 4)$
Let $u = x^2$: $2(u - 1)(u - 4)$.
Replace u and factor each difference of squares.
Final: $2(x - 1)(x + 1)(x - 2)(x + 2)$

4. $x^6 - 64$

Method: Difference of squares, then cubes
$x^6 - 64 = (x^3 - 8)(x^3 + 8)$.
Factor each cube expression.
Final: $(x - 2)(x^2 + 2x + 4)(x + 2)(x^2 - 2x + 4)$

5. $16x^4 - 40x^2y^2 + 25y^4$

Method: Perfect-square trinomial
$16x^4 = (4x^2)^2$ and $25y^4 = (5y^2)^2$.
$-40x^2y^2 = 2(4x^2)(-5y^2)$.
Final: $(4x^2 - 5y^2)^2$

6. $18x^2 - 33x - 30$

Method: GCF, then AC method
Factor out 3: $3(6x^2 - 11x - 10)$.
$ac = -60$. Use -15 and 4.
$6x^2 - 15x + 4x - 10$
Final: $3(3x + 2)(2x - 5)$

7. $4x^4 + 4x^2 - 15$

Method: Quadratic form
Let $u = x^2$: $4u^2 + 4u - 15$.
$(2u - 3)(2u + 5)$
Replace u with x^2.
Final: $(2x^2 - 3)(2x^2 + 5)$

8. $x^4 + 2x^3 - 9x^2 - 18x$

Method: GCF, grouping, then difference of squares

Factor out x.

$x[x^2(x + 2) - 9(x + 2)]$

$x(x + 2)(x^2 - 9)$

Final: $x(x + 2)(x - 3)(x + 3)$

9. $16a^4 - 81b^4$

Method: Difference of squares, twice

$(4a^2 - 9b^2)(4a^2 + 9b^2)$

Factor $4a^2 - 9b^2$ again.

Final: $(2a - 3b)(2a + 3b)(4a^2 + 9b^2)$

10. $3x^5 - 21x^3 + 36x$

Method: GCF, quadratic form, then difference of squares

Factor out 3x: $3x(x^4 - 7x^2 + 12)$.

Let $u = x^2$: $(u - 3)(u - 4)$.

Replace u and factor $x^2 - 4$.

Final: $3x(x^2 - 3)(x - 2)(x + 2)$