

Rational Root Theorem Worksheet

Possible rational zeros -> test candidates -> synthetic division -> finish the factorization

Directions: Level 1: list all possible rational zeros using $\pm p/q$. Level 2: list candidates, test them, and find all rational zeros. Level 3: use the Rational Root Theorem to find rational zeros, divide them out, and then find **all** remaining zeros from the final quadratic.

Quick Candidate Generator

p = factors of constant term

q = factors of leading coefficient

candidates = $\pm p/q$

Reduce duplicate fractions, include both signs, and remember: a candidate becomes a root only if testing gives remainder 0.

Level 1: List Possible Rational Zeros Only

1. $f(x) = x^3 - 6x^2 + 11x - 6$

p factors: _____ q factors: _____ possible roots: _____

2. $f(x) = 2x^3 + 5x^2 - x - 6$

p factors: _____ q factors: _____ possible roots: _____

3. $f(x) = 3x^4 - 5x^2 + 8x - 4$

p factors: _____ q factors: _____ possible roots: _____

4. $f(x) = 4x^3 - 7x + 10$

p factors: _____ q factors: _____ possible roots: _____

5. $f(x) = 5x^4 - 2x^3 + 7x - 3$

p factors: _____ q factors: _____ possible roots: _____

6. $f(x) = 6x^5 + x^2 - 12$

p factors: _____ q factors: _____ possible roots: _____

Level 2: Test Candidates and Find All Rational Zeros

For each polynomial, write the candidate list, test candidates using substitution or synthetic division, and factor completely over the rationals.

7. $f(x) = x^3 - 6x^2 + 11x - 6$

possible rational zeros: _____

test / synthetic division work:

rational zeros: _____

8. $f(x) = 2x^3 - x^2 - 8x + 4$

possible rational zeros: _____

test / synthetic division work:

rational zeros: _____

9. $f(x) = 3x^3 + x^2 - 12x - 4$

possible rational zeros: _____

test / synthetic division work:

rational zeros: _____

10. $f(x) = 2x^3 + x^2 - 18x - 9$

possible rational zeros: _____

test / synthetic division work:

rational zeros: _____

11. $f(x) = 2x^3 - 3x^2 - 3x + 2$

possible rational zeros: _____

test / synthetic division work:

rational zeros: _____

12. $f(x) = x^4 - 5x^3 + 5x^2 + 5x - 6$

possible rational zeros: _____

test / synthetic division work:

rational zeros: _____

Level 3: Rational Roots + Remaining Quadratic

Find the rational zeros first. Divide out confirmed factors. Then solve the remaining quadratic, even if its zeros are irrational or complex.

13. $f(x) = 2x^3 - 5x^2 - 4x + 3$

possible rational zeros: _____

synthetic division / quotient: _____

all zeros: _____

14. $f(x) = 3x^3 - 8x^2 - 5x + 6$

possible rational zeros: _____

synthetic division / quotient: _____

all zeros: _____

15. $f(x) = x^4 - x^3 - x^2 - x - 2$

possible rational zeros: _____

synthetic division / quotient: _____

all zeros: _____

16. $f(x) = 2x^4 + 5x^3 - 7x^2 - 10x + 6$

possible rational zeros: _____

synthetic division / quotient: _____

all zeros: _____

17. $f(x) = x^4 + x^3 - 7x^2 - 5x + 10$

possible rational zeros: _____

synthetic division / quotient: _____

all zeros: _____

18. $f(x) = 2x^4 + x^3 + 7x^2 + 4x - 4$

possible rational zeros: _____

synthetic division / quotient: _____

all zeros: _____

Extension: Choose one Level 3 problem. Explain why the Rational Root Theorem does *not* promise that every zero is rational. Identify which zeros came from p/q candidates and which came from the remaining quadratic.