Rational Root Theorem Cheat Sheet

p/q -> test -> divide -> finish: a one-page reference for finding polynomial zeros

The theorem: If f(x) has integer coefficients and p/q is a rational zero in lowest terms, then p must divide the constant term and q must divide the leading coefficient.

1. Standard form
Use integer coefficients and identify the constant term and leading coefficient.

DOWN

2. Build p
List the factors of the constant term a0.

DOWN

3. Build q
List the factors of the leading coefficient an.

DOWN

4. Generate +/- p/q
Form every possible rational root and reduce duplicates.

DOWN

5. Test candidates
Use substitution or synthetic division. Remainder 0 means the candidate is a root.

DOWN

6. Divide and repeat
Factor out the confirmed root. When the quotient is quadratic, factor or use the quadratic formula.

Worked Candidate Example

f(x) = 2x^3 - 5x^2 - 4x + 3
p factors: +/-1, +/-3 q factors: +/-1, +/-2
possible rational zeros: +/-1, +/-3, +/-1/2, +/-3/2

Test x = 1/2 with synthetic division: remainder = 0, so (2x - 1) is a factor. The quotient is 2x^2 - 4x - 6 = 2(x - 3)(x + 1).
All zeros: -1, 1/2, 3.

Fast Checks That Prevent Common Errors

Use integer coefficients	The theorem is stated for polynomials with integer coefficients.
Include both signs	Every positive p/q candidate also creates a negative candidate.
Reduce duplicates	For example, 2/2 = 1 and 4/2 = 2; list each candidate only once.
Candidate does not mean root	A p/q value is only possible until it gives remainder 0.
Remainder 0 means factor	If synthetic division by c gives remainder 0, then x - c is a factor.
Finish the quotient	Once the quotient is quadratic, factor it or use the quadratic formula.

Rational Root Theorem Worksheet

Possible rational zeros -> test candidates -> synthetic division -> finish the factorization

Directions: Level 1: list all possible rational zeros using +/-p/q. Level 2: list candidates, test them, and find all rational zeros. Level 3: use the Rational Root Theorem to find rational zeros, divide them out, and then find **all** remaining zeros from the final quadratic.

Quick p/q Reference

p = factors of constant term	**q** = factors of leading coefficient	**candidates** = +/-p/q

Reduce duplicate fractions, include both signs, and remember: a candidate becomes a root only if testing gives remainder 0.

Level 1: List Possible Rational Zeros Only

1. f(x) = x^3 - 6x^2 + 11x - 6

p factors: ____________________ q factors: ____________________ possible roots: ____

2. f(x) = 2x^3 + 5x^2 - x - 6

p factors: ____________________ q factors: ____________________ possible roots: ____

3. f(x) = 3x^4 - 5x^2 + 8x - 4

p factors: ____________________ q factors: ____________________ possible roots: ____

4. f(x) = 4x^3 - 7x + 10

p factors: ____________________ q factors: ____________________ possible roots: ____

5. f(x) = 5x^4 - 2x^3 + 7x - 3

p factors: ____________________ q factors: ____________________ possible roots: ____

6. f(x) = 6x^5 + x^2 - 12

p factors: ____________________ q factors: ____________________ possible roots: ____

Level 2: Test Candidates and Find All Rational Zeros

For each polynomial, write the candidate list, test candidates using substitution or synthetic division, and factor completely over the rationals.

7. $f(x) = x^3 - 6x^2 + 11x - 6$

possible rational zeros: ____

test / synthetic division work:

rational zeros: ____

8. $f(x) = 2x^3 - x^2 - 8x + 4$

possible rational zeros: ____

test / synthetic division work:

rational zeros: ____

9. $f(x) = 3x^3 + x^2 - 12x - 4$

possible rational zeros: ____

test / synthetic division work:

rational zeros: ____

10. $f(x) = 2x^3 + x^2 - 18x - 9$

possible rational zeros: ____

test / synthetic division work:

rational zeros: ____

11. $f(x) = 2x^3 - 3x^2 - 3x + 2$

possible rational zeros: ____

test / synthetic division work:

rational zeros: ____

12. $f(x) = x^4 - 5x^3 + 5x^2 + 5x - 6$

possible rational zeros: ____

test / synthetic division work:

rational zeros: ____

Level 3: Rational Roots + Remaining Quadratic

Find the rational zeros first. Divide out confirmed factors. Then solve the remaining quadratic, even if its zeros are irrational or complex.

13. $f(x) = 2x^3 - 5x^2 - 4x + 3$

possible rational zeros: ____

synthetic division / quotient:

all zeros: ____

14. $f(x) = 3x^3 - 8x^2 - 5x + 6$

possible rational zeros: ____

synthetic division / quotient:

all zeros: ____

15. $f(x) = x^4 - x^3 - x^2 - x - 2$

possible rational zeros: ____

synthetic division / quotient:

all zeros: ____

16. $f(x) = 2x^4 + 5x^3 - 7x^2 - 10x + 6$

possible rational zeros: ____

synthetic division / quotient:

all zeros: ____

17. $f(x) = x^4 + x^3 - 7x^2 - 5x + 10$

possible rational zeros: ____

synthetic division / quotient:

all zeros: ____

18. $f(x) = 2x^4 + x^3 + 7x^2 + 4x - 4$

possible rational zeros: ____

synthetic division / quotient:

all zeros: ____

Extension: Choose one Level 3 problem. Explain why the Rational Root Theorem does *not* promise that every zero is rational. Identify which zeros came from p/q candidates and which came from the remaining quadratic.

Rational Root Theorem Worksheet - Answer Key

Candidate lists are reduced to unique +/- values.

Level 1: Possible Rational Zeros

#	Polynomial	Possible rational zeros
1	x^3 - 6x^2 + 11x - 6	+/- 1, +/- 2, +/- 3, +/- 6
2	2x^3 + 5x^2 - x - 6	+/- 1/2, +/- 1, +/- 3/2, +/- 2, +/- 3, +/- 6
3	3x^4 - 5x^2 + 8x - 4	+/- 1/3, +/- 2/3, +/- 1, +/- 4/3, +/- 2, +/- 4
4	4x^3 - 7x + 10	+/- 1/4, +/- 1/2, +/- 1, +/- 5/4, +/- 2, +/- 5/2, +/- 5, +/- 10
5	5x^4 - 2x^3 + 7x - 3	+/- 1/5, +/- 3/5, +/- 1, +/- 3
6	6x^5 + x^2 - 12	+/- 1/6, +/- 1/3, +/- 1/2, +/- 2/3, +/- 1, +/- 4/3, +/- 3/2, +/- 2, +/- 3, +/- 4, +/- 6, +/- 12

Level 2: Rational Zeros

#	Possible rational zeros	Factorization	Rational zeros
7	+/- 1, +/- 2, +/- 3, +/- 6	(x - 1)(x - 2)(x - 3)	1, 2, 3
8	+/- 1/2, +/- 1, +/- 2, +/- 4	(x + 2)(2x - 1)(x - 2)	-2, 1/2, 2
9	+/- 1/3, +/- 2/3, +/- 1, +/- 4/3, +/- 2, +/- 4	(x + 2)(3x + 1)(x - 2)	-2, -1/3, 2
10	+/- 1/2, +/- 1, +/- 3/2, +/- 3, +/- 9/2, +/- 9	(x + 3)(2x + 1)(x - 3)	-3, -1/2, 3
11	+/- 1/2, +/- 1, +/- 2	(x + 1)(2x - 1)(x - 2)	-1, 1/2, 2
12	+/- 1, +/- 2, +/- 3, +/- 6	(x + 1)(x - 1)(x - 2)(x - 3)	-1, 1, 2, 3

Reminder: The candidate list comes from +/-p/q. A value becomes an actual zero only after substitution or synthetic division gives remainder 0.

Level 3: All Zeros After Rational Root Testing

#	Possible rational zeros	Factorization	All zeros
13	+/- 1/2, +/- 1, +/- 3/2, +/- 3	(x + 1)(2x - 1)(x - 3)	-1, 1/2, 3
14	+/- 1/3, +/- 2/3, +/- 1, +/- 2, +/- 3, +/- 6	(x + 1)(3x - 2)(x - 3)	-1, 2/3, 3
15	+/- 1, +/- 2	(x + 1)(x - 2)(x^2 + 1)	-1, 2, -i, i
16	+/- 1/2, +/- 1, +/- 3/2, +/- 2, +/- 3, +/- 6	(x + 3)(2x - 1)(x^2 - 2)	-3, 1/2, -sqrt(2), sqrt(2)
17	+/- 1, +/- 2, +/- 5, +/- 10	(x + 2)(x - 1)(x^2 - 5)	-2, 1, -sqrt(5), sqrt(5)
18	+/- 1/2, +/- 1, +/- 2, +/- 4	(x + 1)(2x - 1)(x^2 + 4)	-1, 1/2, -2i, 2i

Why Level 3 matters: The Rational Root Theorem generates only *possible rational* zeros. After rational factors are divided out, the remaining quadratic can still produce irrational or complex zeros.