Rational Root Theorem Worksheet - Answer Key

Candidate lists are reduced to unique +/- values.

Level 1: Possible Rational Zeros

#	Polynomial	Possible rational zeros
1	x^3 - 6x^2 + 11x - 6	+/- 1, +/- 2, +/- 3, +/- 6
2	2x^3 + 5x^2 - x - 6	+/- 1/2, +/- 1, +/- 3/2, +/- 2, +/- 3, +/- 6
3	3x^4 - 5x^2 + 8x - 4	+/- 1/3, +/- 2/3, +/- 1, +/- 4/3, +/- 2, +/- 4
4	4x^3 - 7x + 10	+/- 1/4, +/- 1/2, +/- 1, +/- 5/4, +/- 2, +/- 5/2, +/- 5, +/- 10
5	5x^4 - 2x^3 + 7x - 3	+/- 1/5, +/- 3/5, +/- 1, +/- 3
6	6x^5 + x^2 - 12	+/- 1/6, +/- 1/3, +/- 1/2, +/- 2/3, +/- 1, +/- 4/3, +/- 3/2, +/- 2, +/- 3, +/- 4, +/- 6, +/- 12

Level 2: Rational Zeros

#	Possible rational zeros	Factorization	Rational zeros
7	+/- 1, +/- 2, +/- 3, +/- 6	(x - 1)(x - 2)(x - 3)	1, 2, 3
8	+/- 1/2, +/- 1, +/- 2, +/- 4	(x + 2)(2x - 1)(x - 2)	-2, 1/2, 2
9	+/- 1/3, +/- 2/3, +/- 1, +/- 4/3, +/- 2, +/- 4	(x + 2)(3x + 1)(x - 2)	-2, -1/3, 2
10	+/- 1/2, +/- 1, +/- 3/2, +/- 3, +/- 9/2, +/- 9	(x + 3)(2x + 1)(x - 3)	-3, -1/2, 3
11	+/- 1/2, +/- 1, +/- 2	(x + 1)(2x - 1)(x - 2)	-1, 1/2, 2
12	+/- 1, +/- 2, +/- 3, +/- 6	(x + 1)(x - 1)(x - 2)(x - 3)	-1, 1, 2, 3

Reminder: The candidate list comes from +/-p/q. A value becomes an actual zero only after substitution or synthetic division gives remainder 0.

Level 3: All Zeros After Rational Root Testing

#	Possible rational zeros	Factorization	All zeros
13	+/- 1/2, +/- 1, +/- 3/2, +/- 3	(x + 1)(2x - 1)(x - 3)	-1, 1/2, 3
14	+/- 1/3, +/- 2/3, +/- 1, +/- 2, +/- 3, +/- 6	(x + 1)(3x - 2)(x - 3)	-1, 2/3, 3
15	+/- 1, +/- 2	(x + 1)(x - 2)(x^2 + 1)	-1, 2, -i, i
16	+/- 1/2, +/- 1, +/- 3/2, +/- 2, +/- 3, +/- 6	(x + 3)(2x - 1)(x^2 - 2)	-3, 1/2, -sqrt(2), sqrt(2)
17	+/- 1, +/- 2, +/- 5, +/- 10	(x + 2)(x - 1)(x^2 - 5)	-2, 1, -sqrt(5), sqrt(5)
18	+/- 1/2, +/- 1, +/- 2, +/- 4	(x + 1)(2x - 1)(x^2 + 4)	-1, 1/2, -2i, 2i

Why Level 3 matters: The Rational Root Theorem generates only *possible rational* zeros. After rational factors are divided out, the remaining quadratic can still produce irrational or complex zeros.