Rational Root Theorem Cheat Sheet

p/q -> test -> divide -> finish: a one-page reference for finding polynomial zeros

The theorem: If f(x) has integer coefficients and p/q is a rational zero in lowest terms, then p must divide the constant term and q must divide the leading coefficient.

1. Standard form
Use integer coefficients and identify the constant term and leading coefficient.

DOWN

2. Build p
List the factors of the constant term a0.

DOWN

3. Build q
List the factors of the leading coefficient an.

DOWN

4. Generate +/- p/q
Form every possible rational root and reduce duplicates.

DOWN

5. Test candidates
Use substitution or synthetic division. Remainder 0 means the candidate is a root.

DOWN

6. Divide and repeat
Factor out the confirmed root. When the quotient is quadratic, factor or use the quadratic formula.

Worked Candidate Example

f(x) = 2x^3 - 5x^2 - 4x + 3
p factors: +/-1, +/-3   q factors: +/-1, +/-2
possible rational zeros: +/-1, +/-3, +/-1/2, +/-3/2

Test x = 1/2 with synthetic division: remainder = 0, so (2x - 1) is a factor. The quotient is 2x^2 - 4x - 6 = 2(x - 3)(x + 1).
All zeros: -1, 1/2, 3.

Fast Checks That Prevent Common Errors

Use integer coefficients	The theorem is stated for polynomials with integer coefficients.
Include both signs	Every positive p/q candidate also creates a negative candidate.
Reduce duplicates	For example, 2/2 = 1 and 4/2 = 2; list each candidate only once.
Candidate does not mean root	A p/q value is only possible until it gives remainder 0.
Remainder 0 means factor	If synthetic division by c gives remainder 0, then x - c is a factor.
Finish the quotient	Once the quotient is quadratic, factor it or use the quadratic formula.