Polynomial Inequalities Sign-Chart Cheat Sheet

Quick reference for factoring, zeros, multiplicity, signs, and interval notation

1. Move to one side Rewrite so the other side is 0.	2. Factor completely Find all factors and repeated roots.	3. Mark the zeros Zeros split the number line into intervals.	4. Find each sign Test one value or reason from factor signs.	5. Choose intervals Keep only intervals satisfying the inequality.

Multiplicity: does the sign change?

Odd multiplicity: the graph crosses the x-axis, so the sign changes.

Even multiplicity: the graph touches the x-axis and turns, so the sign stays the same.

Example: in $(x - 2)^2(x + 3)$, $x = 2$ has even multiplicity.

Endpoint and interval notation rules

$>$ or $<$ → exclude zeros → parentheses

$\geq$ or $\leq$ → include zeros that satisfy equality → brackets

∞ and $-\infty$ always use parentheses.

Worked sign-chart example

Solve: $(x + 3)(x - 1)(x - 4) \geq 0$ Zeros: -3, 1, 4

Interval	**$(-\infty, -3)$**	**$(-3, 1)$**	**$(1, 4)$**	**$(4, \infty)$**
Sign	Negative	Positive	Negative	Positive
Use for ≥ 0?	No	Yes	No	Yes

Answer: $[-3, 1] \cup [4, \infty)$. Each zero has odd multiplicity, so the sign changes at every zero.

Fast sign patterns to remember

Two simple zeros, positive leading coefficient: positive outside, negative between.

Odd-degree polynomial with simple ordered zeros: signs alternate as you pass each zero.

Repeated even zero: do not flip the sign at that zero.

Final check before submitting

Did you factor completely? Did you include every zero? Did you account for multiplicity?

Did you use brackets only when equality is allowed? Did you keep parentheses beside infinity?

Did you give the answer in interval notation rather than only listing test points?

Repeated-root mini example

Solve: $(x + 1)^2(x - 4) < 0$ Zeros: -1 (even multiplicity), 4 (odd multiplicity)

$(-\infty, -1)$ Negative → use	$x = -1$ Zero → exclude	$(-1, 4)$ Negative → use	$x = 4$ Zero → exclude	$(4, \infty)$ Positive → do not use

Answer: $(-\infty, -1) \cup (-1, 4)$. The sign does not flip at $x = -1$ because the factor is squared.

Interval-notation snapshots

$x < -2$ or $x > 5$ $(-\infty, -2) \cup (5, \infty)$	$-1 \leq x \leq 4$ $[-1, 4]$	$x = -2$ or $1 \leq x \leq 4$ $\{-2\} \cup [1, 4]$

Polynomial Inequalities Worksheet

Sign charts, factoring, interval notation, graph interpretation, and three difficulty levels

Name: ______________________________ Date: ____________________

1. Rewrite	2. Factor	3. Find zeros	4. Test signs	5. Write intervals
Put 0 on one side.	Factor completely.	Use zeros as boundaries.	Test each interval or use multiplicity.	Match >, <, ≥, or ≤.

Quick sign-chart reference

For (x + 3)(x - 1)(x - 4), the zeros -3, 1, and 4 divide the number line into four intervals. Since each zero has odd multiplicity, the sign changes at every zero.

Interval	(-∞, -3)	(-3, 1)	(1, 4)	(4, ∞)
Sign	Negative	Positive	Negative	Positive
Use for p(x) ≥ 0?	No	Yes	No	Yes

Reference solution: (x + 3)(x - 1)(x - 4) ≥ 0 gives [-3, 1] ∪ [4, ∞).

Level 1 - Build the Sign Chart

These expressions are already factored. Find the zeros, determine the sign on each interval, and write the solution in interval notation.

1. $(x - 2)(x + 5) > 0$

2. $(x + 1)(x - 4) \leq 0$

3. $(x - 3)(x + 2)(x + 6) \geq 0$

4. $(x + 4)(x - 1)(x - 5) < 0$

5. $(x - 2)^2(x + 3) \geq 0$

6. Graph interpretation: A quadratic has x-intercepts at -4 and 2 and opens upward. On what interval is $f(x) \leq 0$?

Show your sign chart or number-line work in the space below.

Level 2 - Factor First

Factor each polynomial completely before building a sign chart. Show the factorization and write the final solution in interval notation.

7. $x^2 - 7x + 10 > 0$

8. $x^2 + x - 12 \leq 0$

9. $x^3 - 4x^2 - x + 4 \geq 0$

10. $x^3 + x^2 - 9x - 9 < 0$

11. $2x^2 - 5x - 3 \geq 0$

12. Graph interpretation: A polynomial crosses the x-axis at $x = -3, 1$, and 4. The graph is below the x-axis on $(-\infty, -3)$ and $(1, 4)$. Solve $f(x) < 0$.

Factoring workspace

Level 3 - Multiplicity and Higher Degree

Track repeated zeros carefully. Odd multiplicity changes the sign; even multiplicity keeps the same sign on both sides of the zero.

13. $(x - 3)^2(x + 1)(x - 5) > 0$

14. $(x + 2)^2(x - 1)(x - 4) \le 0$

15. $(x^2 - 9)(x - 2) \ge 0$

16. $x^4 - 5x^2 + 4 \ge 0$

17. $x^4 + x^3 - 7x^2 - x + 6 < 0$

18. Graph interpretation: A polynomial touches the x-axis at $x = -2$, crosses at $x = 1$ and $x = 5$, lies above the x-axis for $x < 1$, below it on $(1, 5)$, and above it again for $x > 5$. Solve $f(x) \ge 0$.

Optional extension

Choose any two problems and sketch a possible polynomial graph that matches your sign chart. Show the x-intercepts and indicate whether the graph crosses or touches the x-axis at each repeated zero.

Polynomial Inequalities Worksheet

Complete answer key with factoring notes and interval-notation solutions

This key follows the same 18 problems as the student worksheet. Equivalent interval-notation answers are acceptable.

Level 1 Answers

#	Reasoning / factorization	Solution
1	Critical values: -5, 2. Signs are +, -, +.	(-∞, -5) ∪ (2, ∞)
2	Critical values: -1, 4. The product is nonpositive between the zeros.	[-1, 4]
3	Critical values: -6, -2, 3. Signs are -, +, -, +.	[-6, -2] ∪ [3, ∞)
4	Critical values: -4, 1, 5. Signs are -, +, -, +.	(-∞, -4) ∪ (1, 5)
5	Zeros: -3 and 2. The zero at 2 has even multiplicity, so the sign does not change there.	[-3, ∞)
6	An upward-opening quadratic is at or below the x-axis between its intercepts.	[-4, 2]

Level 2 Answers

#	Reasoning / factorization	Solution
7	Factor: $(x - 2)(x - 5) > 0$.	$(-\infty, 2) \cup (5, \infty)$
8	Factor: $(x + 4)(x - 3) \leq 0$.	$[-4, 3]$
9	Factor by grouping: $(x - 4)(x - 1)(x + 1) \geq 0$.	$[-1, 1] \cup [4, \infty)$
10	Factor by grouping: $(x - 3)(x + 1)(x + 3) < 0$.	$(-\infty, -3) \cup (-1, 3)$
11	Factor: $(2x + 1)(x - 3) \geq 0$.	$(-\infty, -1/2] \cup [3, \infty)$
12	Read the intervals where the graph is below the x-axis; strict inequality excludes zeros.	$(-\infty, -3) \cup (1, 4)$

Level 3 Answers

#	Reasoning / factorization	Solution
13	The factor $(x - 3)^2$ has even multiplicity. Sign is controlled by $(x + 1)(x - 5)$, except $x = 3$ is a zero.	$(-\infty, -1) \cup (5, \infty)$
14	The product is negative on (1, 4), zero at 1 and 4, and also zero at the isolated even-multiplicity root $x = -2$.	$\{-2\} \cup [1, 4]$
15	Factor: $(x + 3)(x - 3)(x - 2) \geq 0$.	$[-3, 2] \cup [3, \infty)$
16	Factor: $(x^2 - 1)(x^2 - 4) = (x - 1)(x + 1)(x - 2)(x + 2)$.	$(-\infty, -2] \cup [-1, 1] \cup [2, \infty)$
17	Factor: $(x - 2)(x - 1)(x + 1)(x + 3) < 0$.	$(-3, -1) \cup (1, 2)$
18	The graph is nonnegative for $x < 1$, equals 0 at $x = 1$, is negative on (1, 5), and is nonnegative again from $x = 5$ onward.	$(-\infty, 1] \cup [5, \infty)$

Accuracy note

The polynomial factorizations and inequality solution sets in this answer key were checked before publication. Repeated-root problems were verified using multiplicity-based sign analysis.