

Descartes' Rule of Signs Cheat Sheet

Build a complete root profile: positive -> negative -> nonreal

1. Put $f(x)$ in descending powers

Ignore zero coefficients when reading the sign pattern.

2. Count sign changes in $f(x)$

If $V+ = 3$, the possible positive real zero counts are 3 or 1.

3. Find $f(-x)$ and count again

Odd-power terms change sign; even-power terms do not. If $V- = 2$, negative zeros are 2 or 0.

4. Combine the possibilities

Pair every allowed positive count with every allowed negative count.

5. Finish with nonreal zeros

nonreal = degree - positive - negative. For real coefficients, the nonreal count must be even.

6. Watch for $x = 0$

If the constant term is 0, factor out x^m first. Zero roots are separate from positive/negative counts.

Worked example

$$f(x) = x^5 - 3x^4 - 2x^3 + 6x^2 + 5x - 7$$

A

Sign pattern of $f(x)$: + - - + + -

3 sign changes -> positive roots: 3 or 1

B

$$f(-x) = -x^5 - 3x^4 + 2x^3 + 6x^2 - 5x - 7$$

Sign pattern: - - + + - - -> 2 changes -> negative roots: 2 or 0

C

Combine positive and negative possibilities

(3,2), (3,0), (1,2), (1,0)

D

Use degree 5 to finish the profile

(3 positive, 2 negative, 0 nonreal); (3 positive, 0 negative, 2 nonreal); (1 positive, 2 negative, 2 nonreal); (1 positive, 0 negative, 4 nonreal)

Memory rule: sign changes give a maximum. Subtract 2, then 2 again, until you reach 0 or 1. Descartes' Rule counts multiplicity.

Descartes' Rule of Signs Worksheet

Positive roots -> negative roots -> nonreal roots | Algebra 2 & Precalculus

Directions: Write polynomials in descending powers. Ignore zero coefficients when counting sign changes. For positive roots, count sign changes in $f(x)$. For negative roots, count sign changes in $f(-x)$. Possible counts decrease by 2. Use the degree to determine how many nonreal zeros may remain.

Level 1 - Sign Changes & Real Zeros

1. $f(x) = x^4 - 3x^3 + 2x^2 - x + 5$ Count sign changes in $f(x)$ and $f(-x)$. State the possible numbers of positive and negative real zeros.	Work:
2. $f(x) = 2x^5 + x^4 - 4x^3 - 3x^2 + 6x + 1$ Count sign changes in $f(x)$ and $f(-x)$. State the possible numbers of positive and negative real zeros.	Work:
3. $f(x) = x^5 - 2x^4 + 5x^2 - 7$ Count sign changes in $f(x)$ and $f(-x)$. State the possible numbers of positive and negative real zeros.	Work:
4. $f(x) = -x^4 + 3x^3 + 2x^2 - 4x - 8$ Count sign changes in $f(x)$ and $f(-x)$. State the possible numbers of positive and negative real zeros.	Work:
5. $f(x) = x^6 + 2x^5 - 3x^4 - 5x^2 + 4x + 6$ Count sign changes in $f(x)$ and $f(-x)$. State the possible numbers of positive and negative real zeros.	Work:
6. $f(x) = x^4 + 4x^3 + 6x^2 + 4x + 1$ Count sign changes in $f(x)$ and $f(-x)$. State the possible numbers of positive and negative real zeros.	Work:

Level 2 - Complete the Root Count

Add the possible number of nonreal zeros after finding the positive and negative possibilities.

7. $f(x) = x^5 - 3x^4 - 2x^3 + 6x^2 + 5x - 7$ Find possible positive, negative, and nonreal zero counts.	Work:
8. $f(x) = -x^5 + 2x^4 - 3x^2 + 5x - 6$ Find possible positive, negative, and nonreal zero counts.	Work:
9. $f(x) = 3x^4 - 2x^3 - 5x^2 + 7x + 1$ Find possible positive, negative, and nonreal zero counts.	Work:
10. $f(x) = x^4 - 5x^2 + 4x - 2$ Find possible positive, negative, and nonreal zero counts.	Work:
11. $f(x) = 2x^6 - x^5 + 3x^4 - 4x^3 + 5x^2 - 6x + 7$ Find possible positive, negative, and nonreal zero counts.	Work:
12. $f(x) = x^5 + 5x^4 - 2x^3 - 8x^2 + 3x + 9$ Find possible positive, negative, and nonreal zero counts.	Work:

Level 3 - Root Profile Challenge

List every valid (positive, negative, nonreal) profile. Remember that nonreal zeros of real-coefficient polynomials occur in conjugate pairs.

13. $f(x) = x^4 - 4x^3 + 3x^2 - 2x + 5$ List every possible root profile: positive real, negative real, and nonreal zeros.	Work:
14. $f(x) = 2x^5 + 3x^4 - 5x^3 + 4x - 6$ List every possible root profile: positive real, negative real, and nonreal zeros.	Work:
15. $f(x) = x^6 - x^5 - 6x^4 + 6x^3 + 5x^2 - 5x + 2$ List every possible root profile: positive real, negative real, and nonreal zeros.	Work:
16. $f(x) = -2x^5 + 5x^4 - 3x^3 + 4x - 1$ List every possible root profile: positive real, negative real, and nonreal zeros.	Work:
17. $f(x) = x^7 - 2x^6 + 3x^4 - 4x^3 + 5x - 6$ List every possible root profile: positive real, negative real, and nonreal zeros.	Work:
18. $f(x) = x^7 - 2x^6 + 3x^4 - 4x^3 + 5x^2$ List every possible root profile: positive real, negative real, and nonreal zeros. Factor out any zero roots first.	Work:

Checkpoint: Descartes' Rule gives possible counts, not the actual root values. A zero root is neither positive nor negative; factor x first when the constant term is 0.

Descartes' Rule of Signs Worksheet - Answer Key

All 18 problems checked for sign changes, positive/negative possibilities, and root profiles.

Level 1

#	V+ / Positive	V- / Negative	Nonreal / profiles
1	4 -> 4, 2, 0	0 -> 0	Use degree to finish nonreal count if asked.
2	2 -> 2, 0	3 -> 3, 1	Use degree to finish nonreal count if asked.
3	3 -> 3, 1	2 -> 2, 0	Use degree to finish nonreal count if asked.
4	2 -> 2, 0	2 -> 2, 0	Use degree to finish nonreal count if asked.
5	2 -> 2, 0	2 -> 2, 0	Use degree to finish nonreal count if asked.
6	0 -> 0	4 -> 4, 2, 0	Use degree to finish nonreal count if asked.

Level 2

#	V+ / Positive	V- / Negative	Nonreal / profiles
7	3 -> 3, 1	2 -> 2, 0	Possible nonreal: 0, 2, 4
8	4 -> 4, 2, 0	1 -> 1	Possible nonreal: 0, 2, 4
9	2 -> 2, 0	2 -> 2, 0	Possible nonreal: 0, 2, 4
10	3 -> 3, 1	1 -> 1	Possible nonreal: 0, 2
11	6 -> 6, 4, 2, 0	0 -> 0	Possible nonreal: 0, 2, 4, 6
12	2 -> 2, 0	3 -> 3, 1	Possible nonreal: 0, 2, 4

Level 3

#	V+ / Positive	V- / Negative	Nonreal / profiles
13	4 -> 4, 2, 0	0 -> 0	(positive 4, negative 0, nonreal 0); (positive 2, negative 0, nonreal 2); (positive 0, negative 0, nonreal 4)
14	3 -> 3, 1	2 -> 2, 0	(positive 3, negative 2, nonreal 0); (positive 3, negative 0, nonreal 2); (positive 1, negative 2, nonreal 2); (positive 1, negative 0, nonreal 4)
15	4 -> 4, 2, 0	2 -> 2, 0	(positive 4, negative 2, nonreal 0); (positive 4, negative 0, nonreal 2); (positive 2, negative 2, nonreal 2); (positive 2, negative 0, nonreal 4); (positive 0, negative 2, nonreal 4); (positive 0, negative 0, nonreal 6)
16	4 -> 4, 2, 0	1 -> 1	(positive 4, negative 1, nonreal 0); (positive 2, negative 1, nonreal 2); (positive 0, negative 1, nonreal 4)
17	5 -> 5, 3, 1	2 -> 2, 0	(positive 5, negative 2, nonreal 0); (positive 5, negative 0, nonreal 2); (positive 3, negative 2, nonreal 2); (positive 3, negative 0, nonreal 4); (positive 1, negative 2, nonreal 4); (positive 1, negative 0, nonreal 6)
18	4 -> 4, 2, 0	1 -> 1	Factor x^2 first. (positive 4, negative 1, nonreal 0, zero 2); (positive 2, negative 1, nonreal 2, zero 2); (positive 0, negative 1, nonreal 4, zero 2)