

Descartes' Rule of Signs Cheat Sheet

Build a complete root profile: positive -> negative -> nonreal

1. Put $f(x)$ in descending powers
Ignore zero coefficients when reading the sign pattern.

2. Count sign changes in $f(x)$
If $V+ = 3$, the possible positive real zero counts are 3 or 1.

3. Find $f(-x)$ and count again
Odd-power terms change sign; even-power terms do not. If $V- = 2$, negative zeros are 2 or 0.

4. Combine the possibilities
Pair every allowed positive count with every allowed negative count.

5. Finish with nonreal zeros
nonreal = degree - positive - negative. For real coefficients, the nonreal count must be even.

6. Watch for $x = 0$
If the constant term is 0, factor out x^m first. Zero roots are separate from positive/negative counts.

Worked example

$$f(x) = x^5 - 3x^4 - 2x^3 + 6x^2 + 5x - 7$$

A Sign pattern of $f(x)$: + - - + + -
3 sign changes -> positive roots: 3 or 1

B $f(-x) = -x^5 - 3x^4 + 2x^3 + 6x^2 - 5x - 7$
Sign pattern: - - + + - - -> 2 changes -> negative roots: 2 or 0

C Combine positive and negative possibilities
(3,2), (3,0), (1,2), (1,0)

D Use degree 5 to finish the profile
(3 positive, 2 negative, 0 nonreal); (3 positive, 0 negative, 2 nonreal); (1 positive, 2 negative, 2 nonreal); (1 positive, 0 negative, 4 nonreal)

Memory rule: sign changes give a maximum. Subtract 2, then 2 again, until you reach 0 or 1. Descartes' Rule counts multiplicity.