Complex Zeros & Conjugate Root Theorem Cheat Sheet

From a non-real zero to a real-coefficient polynomial

Conjugate Root Theorem

If a + bi (b != 0) is a zero of a polynomial with real coefficients, then a - bi must also be a zero. The conjugate zeros occur with the same multiplicity.

Pair the conjugates before expanding

$$(x - (a + bi))(x - (a - bi)) = (x - a)^2 + b^2$$

$$= x^2 - 2ax + (a^2 + b^2)$$

Worked example: Missing Polynomial From Its Zeros

Given zeros: 2, -3, 1 + 2i

1. **Complete the zero set**

   Because the coefficients are real, add the conjugate 1 - 2i.

2. **Write one factor for each zero**

   (x - 2)(x + 3)(x - (1 + 2i))(x - (1 - 2i))

3. **Pair the conjugate factors**

   (x - (1 + 2i))(x - (1 - 2i)) = (x - 1)^2 + 4 = x^2 - 2x + 5

4. **Combine the real factors**

   (x - 2)(x + 3)(x^2 - 2x + 5)

5. **Expand and check the degree**

   x^4 - x^3 - 3x^2 + 17x - 30. Degree 4 matches the four zeros.

Common mistake

The conjugate rule requires real coefficients. Do not add a conjugate automatically when the polynomial is allowed to have non-real coefficients.

Complex Zeros of Polynomial Functions Worksheet

Conjugate Root Theorem | Algebra 2 & AP Precalculus

LEVEL 1 - Conjugates & Degree

Use the conjugate-root theorem. Count multiplicity when finding the least possible degree.

1 A polynomial has real coefficients and 3 + 4i is a zero. What other zero must occur?

2 A polynomial has real coefficients and -2 - 5i is a zero. What other zero must occur?

3 A polynomial has real coefficients and 6i is a zero. What other zero must occur?

4 Known zeros are 2 and -1 + 3i. Complete the zero set and state the least possible degree.

5 Known zeros are -4 (multiplicity 2) and 2 + i. Complete the zero set and state the least possible degree.

6 Known zeros are 1 + 2i (multiplicity 2) and -3. Complete the zero set and state the least possible degree.

Complex Zeros of Polynomial Functions Worksheet

Conjugate Root Theorem | Algebra 2 & AP Precalculus

LEVEL 2 - Build the Polynomial

Assume real coefficients. Add any missing conjugate, build factors, and expand to standard form.

7 Write the least-degree monic polynomial with real coefficients and zeros 2, -3, and 1 + 2i. Give standard form.

8 Write the least-degree monic polynomial with real coefficients and zeros -1 and 4 + i. Give standard form.

9 Write the least-degree monic polynomial with real coefficients and zeros 0 and -2 + 3i. Give standard form.

10 Write the least-degree monic polynomial with real coefficients and zeros 3 (multiplicity 2) and -1 + 2i. Give standard form.

11 Write the least-degree monic polynomial with real coefficients and zeros -2 and 1 + i. Give standard form.

12 Write the least-degree monic polynomial with real coefficients and zeros 5 and 2i. Give standard form.

Complex Zeros of Polynomial Functions Worksheet

Conjugate Root Theorem | Algebra 2 & AP Precalculus

LEVEL 3 - Find All Zeros / Challenge

Use the given complex zero and its conjugate, then factor the remaining polynomial. Show exact answers.

13 Find all zeros of x^3 - 2x^2 + 9x - 18, given that 3i is a zero.

14 Find all zeros of x^3 + x^2 - x + 15, given that 1 + 2i is a zero.

15 Find all zeros of x^4 + 2x^3 - 7x^2 + 2x - 8, given that i is a zero.

16 Find all zeros of x^4 + 2x^3 - 2x^2 - 6x + 5, given that -2 + i is a zero.

17 Find all zeros of x^5 - 7x^4 + 21x^3 - 23x^2 - 52x, given that 2 + 3i is a zero.

18 A real-coefficient polynomial has zeros -2, 1 (multiplicity 2), and 3 + 2i, with leading coefficient 2. Write it in standard form.

Complex Zeros Worksheet - Answer Key

Conjugate Root Theorem | Exact answers

LEVEL 1 - Conjugates & Degree

1. **3 - 4i**

Conjugate pair: 3 + 4i and 3 - 4i.

2. **-2 + 5i**

Change only the sign of the imaginary part.

3. **-6i**

The conjugate of 0 + 6i is 0 - 6i.

4. **2, -1 + 3i, -1 - 3i; degree 3**

One real zero plus one conjugate pair gives 3 zeros.

5. **-4 (mult. 2), 2 + i, 2 - i; degree 4**

The conjugate has the same multiplicity as its partner; here each complex zero occurs once.

6. **-3, 1 + 2i (mult. 2), 1 - 2i (mult. 2); degree 5**

Conjugate roots have matching multiplicity for real-coefficient polynomials.

Complex Zeros Worksheet - Answer Key

Conjugate Root Theorem | Exact answers

LEVEL 2 - Build the Polynomial

7. **x^4 - x^3 - 3x^2 + 17x - 30**

(x - 2)(x + 3)[(x - 1)^2 + 4] = (x - 2)(x + 3)(x^2 - 2x + 5).

8. **x^3 - 7x^2 + 9x + 17**

(x + 1)[(x - 4)^2 + 1] = (x + 1)(x^2 - 8x + 17).

9. **x^3 + 4x^2 + 13x**

x[(x + 2)^2 + 9] = x(x^2 + 4x + 13).

10. **x^4 - 4x^3 + 2x^2 - 12x + 45**

(x - 3)^2[(x + 1)^2 + 4] = (x - 3)^2(x^2 + 2x + 5).

11. **x^3 - 2x + 4**

(x + 2)[(x - 1)^2 + 1] = (x + 2)(x^2 - 2x + 2).

12. **x^3 - 5x^2 + 4x - 20**

(x - 5)(x^2 + 4).

Complex Zeros Worksheet - Answer Key

Conjugate Root Theorem | Exact answers

LEVEL 3 - Find All Zeros / Challenge

13. **2, 3i, -3i**

(x - 2)(x^2 + 9).

14. **-3, 1 + 2i, 1 - 2i**

(x + 3)(x^2 - 2x + 5).

15. **-4, 2, i, -i**

(x + 4)(x - 2)(x^2 + 1).

16. **1 (multiplicity 2), -2 + i, -2 - i**

(x - 1)^2(x^2 + 4x + 5).

17. **0, -1, 4, 2 + 3i, 2 - 3i**

x(x + 1)(x - 4)(x^2 - 4x + 13).

18. **2x^5 - 12x^4 + 20x^3 + 40x^2 - 102x + 52**

2(x + 2)(x - 1)^2[(x - 3)^2 + 4] = 2(x + 2)(x - 1)^2(x^2 - 6x + 13).