Complex Zeros & Conjugate Root Theorem Cheat Sheet

From a non-real zero to a real-coefficient polynomial

Conjugate Root Theorem

If $a + bi$ ($b \neq 0$) is a zero of a polynomial with real coefficients, then $a - bi$ must also be a zero. The conjugate zeros occur with the same multiplicity.

Pair the conjugates before expanding

$$(x - (a + bi))(x - (a - bi)) = (x - a)^2 + b^2$$

$$= x^2 - 2ax + (a^2 + b^2)$$

Worked example: Missing Polynomial From Its Zeros

Given zeros: $2, -3, 1 + 2i$

1. **Complete the zero set**

   Because the coefficients are real, add the conjugate $1 - 2i$.

2. **Write one factor for each zero**

   $(x - 2)(x + 3)(x - (1 + 2i))(x - (1 - 2i))$

3. **Pair the conjugate factors**

   $(x - (1 + 2i))(x - (1 - 2i)) = (x - 1)^2 + 4 = x^2 - 2x + 5$

4. **Combine the real factors**

   $(x - 2)(x + 3)(x^2 - 2x + 5)$

5. **Expand and check the degree**

   $x^4 - x^3 - 3x^2 + 17x - 30$. Degree 4 matches the four zeros.

Common mistake

The conjugate rule requires real coefficients. Do not add a conjugate automatically when the polynomial is allowed to have non-real coefficients.