

Vertex and Axis of Symmetry Worksheet

Worked examples, 20 practice problems, and a complete answer key

$$\text{Axis of symmetry: } x = -b / (2a) \quad \text{or} \quad x = (r_1 + r_2) / 2$$

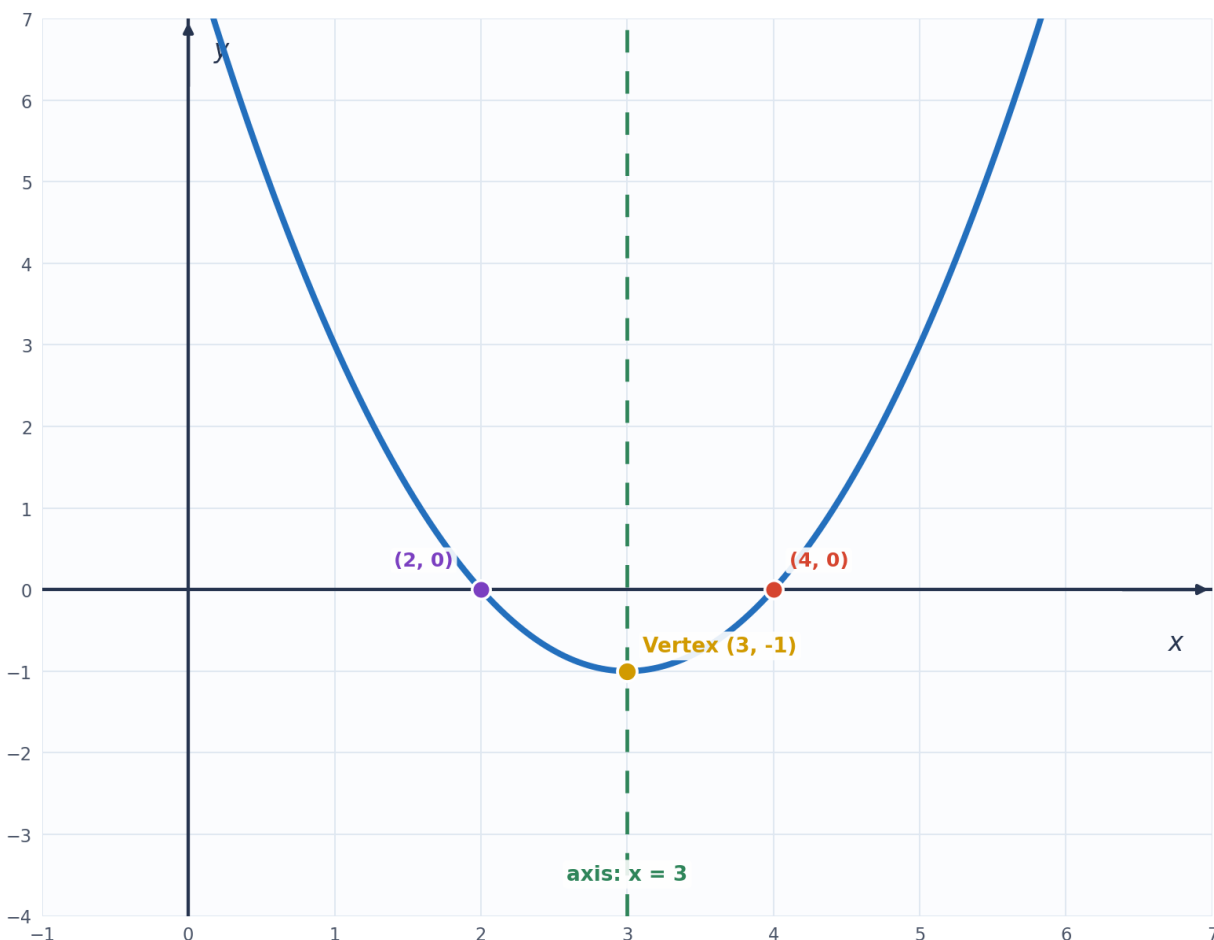
Quick Guide

1. Write the quadratic in standard form: $y = ax^2 + bx + c$.
2. Read a and b with their signs.
3. Compute $x = -b / (2a)$. This x -value is the axis of symmetry.
4. Substitute x into the original equation to find y . The vertex is (x, y) .

Worked Example 1 - Average the Roots

$$y = x^2 - 6x + 8 = (x - 2)(x - 4)$$

The roots are 2 and 4, so the axis is their midpoint: $(2 + 4) / 2 = 3$. Substitute $x = 3$: $y = 9 - 18 + 8 = -1$.

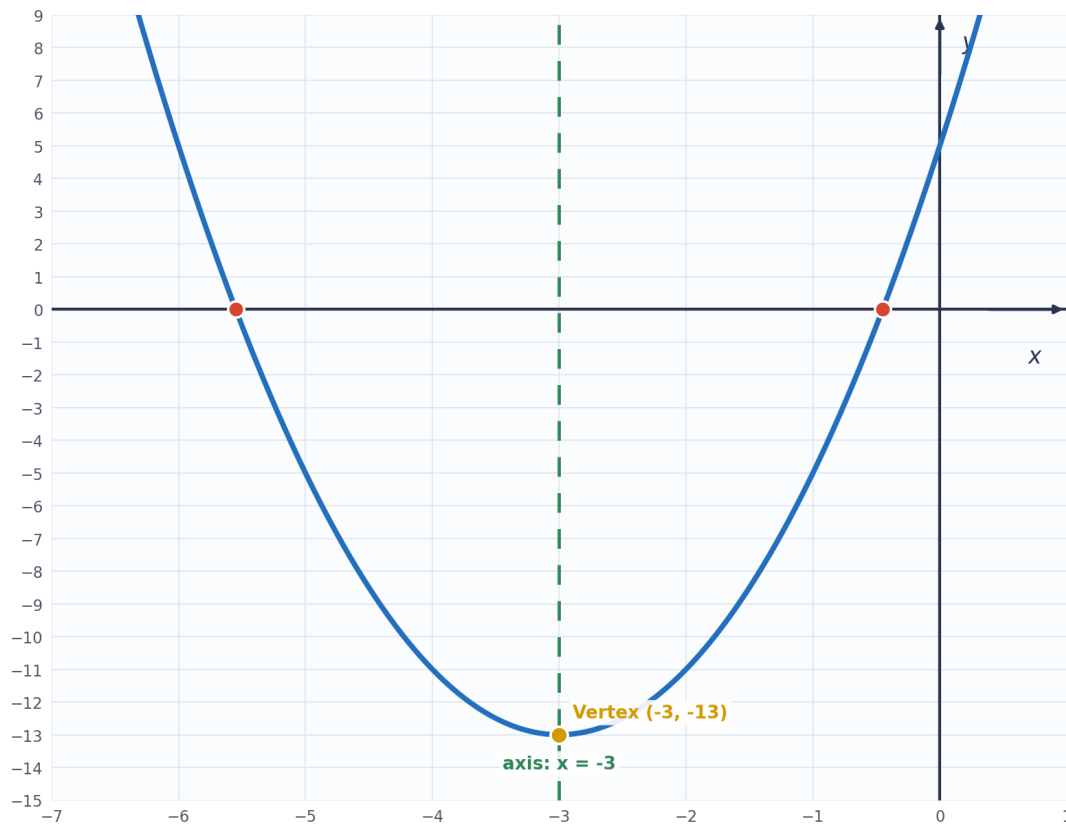


$$\text{Axis of symmetry: } x = 3 \quad \text{Vertex: } (3, -1)$$

Worked Example 2 - Use $x = -b / (2a)$

$$y = 2x^2 + 12x + 5$$

$a = 2$ and $b = 12$, so $x = -12 / 4 = -3$. Substitute $x = -3$: $y = 18 - 36 + 5 = -13$.

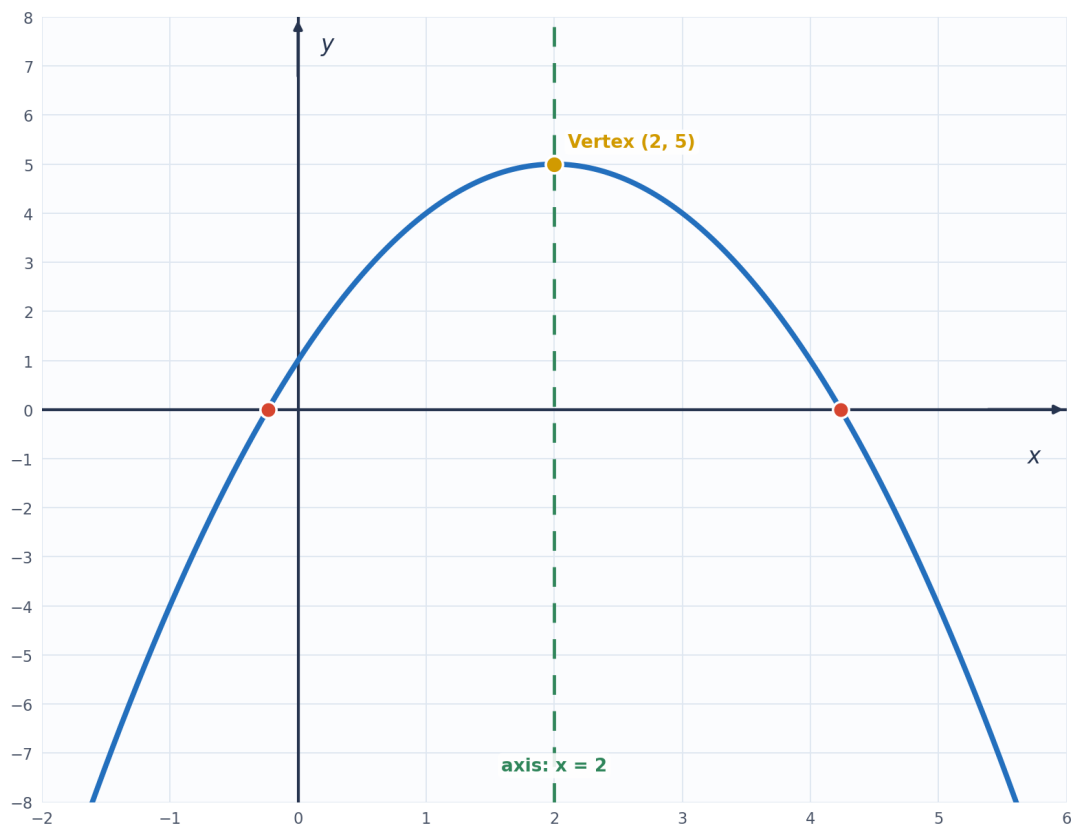


Axis of symmetry: $x = -3$ Vertex: $(-3, -13)$. The vertex is a minimum because $a > 0$.

Worked Example 3 - Negative Leading Coefficient

$$y = -x^2 + 4x + 1$$

$a = -1$ and $b = 4$, so $x = -4 / -2 = 2$. Substitute $x = 2$: $y = -4 + 8 + 1 = 5$.



Axis of symmetry: $x = 2$ Vertex: (2, 5). The vertex is a maximum because $a < 0$.

Practice Problems

For each quadratic, find the axis of symmetry and the vertex. Show your work.

1. $y = x^2 - 4x + 3$

Axis: _____ Vertex: _____

2. $y = x^2 + 6x + 5$

Axis: _____ Vertex: _____

3. $y = x^2 - 2x - 8$

Axis: _____ Vertex: _____

4. $y = 2x^2 + 8x + 3$

Axis: _____ Vertex: _____

5. $y = x^2 + 10x + 21$

Axis: _____ Vertex: _____

Practice Problems - Continued

6. $y = -x^2 + 6x - 5$

Axis: _____ Vertex: _____

7. $y = 3x^2 - 12x + 7$

Axis: _____ Vertex: _____

8. $y = x^2 - 9$

Axis: _____ Vertex: _____

9. $y = x^2 + 5x + 6$

Axis: _____ Vertex: _____

10. $y = x^2 - 8x + 20$

Axis: _____ Vertex: _____

Practice Problems - Continued

11. $y = 4x^2 + 4x - 3$

Axis: _____ Vertex: _____

12. $y = -2x^2 + 8x - 3$

Axis: _____ Vertex: _____

13. $y = x^2 + 3x - 10$

Axis: _____ Vertex: _____

14. $y = x^2 + 2x + 7$

Axis: _____ Vertex: _____

15. $y = 2x^2 - 10x + 8$

Axis: _____ Vertex: _____

Practice Problems - Continued

16. $y = x^2 - 12x + 36$

Axis: _____ Vertex: _____

17. $y = -3x^2 + 6x + 2$

Axis: _____ Vertex: _____

18. $y = x^2 - 7x + 12$

Axis: _____ Vertex: _____

19. $y = 5x^2 + 20x + 11$

Axis: _____ Vertex: _____

20. $y = x^2 + x - 6$

Axis: _____ Vertex: _____

Answer Key

Each solution shows the axis calculation and the substituted y-value.

$$\begin{aligned}1. y &= x^2 - 4x + 3 \\x &= -(-4) / (2(1)) = 2 \\y &= 1(2)^2 + -4(2) + 3 = -1 \\ \text{Axis: } x &= 2 \quad \text{Vertex: } (2, -1)\end{aligned}$$

$$\begin{aligned}2. y &= x^2 + 6x + 5 \\x &= -(6) / (2(1)) = -3 \\y &= 1(-3)^2 + 6(-3) + 5 = -4 \\ \text{Axis: } x &= -3 \quad \text{Vertex: } (-3, -4)\end{aligned}$$

$$\begin{aligned}3. y &= x^2 - 2x - 8 \\x &= -(-2) / (2(1)) = 1 \\y &= 1(1)^2 + -2(1) + -8 = -9 \\ \text{Axis: } x &= 1 \quad \text{Vertex: } (1, -9)\end{aligned}$$

$$\begin{aligned}4. y &= 2x^2 + 8x + 3 \\x &= -(8) / (2(2)) = -2 \\y &= 2(-2)^2 + 8(-2) + 3 = -5 \\ \text{Axis: } x &= -2 \quad \text{Vertex: } (-2, -5)\end{aligned}$$

$$\begin{aligned}5. y &= x^2 + 10x + 21 \\x &= -(10) / (2(1)) = -5 \\y &= 1(-5)^2 + 10(-5) + 21 = -4 \\ \text{Axis: } x &= -5 \quad \text{Vertex: } (-5, -4)\end{aligned}$$

$$\begin{aligned}6. y &= -x^2 + 6x - 5 \\x &= -(6) / (2(-1)) = 3 \\y &= -1(3)^2 + 6(3) + -5 = 4 \\ \text{Axis: } x &= 3 \quad \text{Vertex: } (3, 4) \text{ (maximum)}\end{aligned}$$

$$\begin{aligned}7. y &= 3x^2 - 12x + 7 \\x &= -(-12) / (2(3)) = 2 \\y &= 3(2)^2 + -12(2) + 7 = -5 \\ \text{Axis: } x &= 2 \quad \text{Vertex: } (2, -5)\end{aligned}$$

Answer Key - Continued

$$8. y = x^2 - 9$$

$$x = -(0) / (2(1)) = 0$$

$$y = 1(0)^2 + 0(0) - 9 = -9$$

$$\text{Axis: } x = 0 \quad \text{Vertex: } (0, -9)$$

$$9. y = x^2 + 5x + 6$$

$$x = -(5) / (2(1)) = -5/2$$

$$y = 1(-5/2)^2 + 5(-5/2) + 6 = -1/4$$

$$\text{Axis: } x = -5/2 \quad \text{Vertex: } (-5/2, -1/4)$$

$$10. y = x^2 - 8x + 20$$

$$x = -(-8) / (2(1)) = 4$$

$$y = 1(4)^2 - 8(4) + 20 = 4$$

$$\text{Axis: } x = 4 \quad \text{Vertex: } (4, 4)$$

$$11. y = 4x^2 + 4x - 3$$

$$x = -(4) / (2(4)) = -1/2$$

$$y = 4(-1/2)^2 + 4(-1/2) - 3 = -4$$

$$\text{Axis: } x = -1/2 \quad \text{Vertex: } (-1/2, -4)$$

$$12. y = -2x^2 + 8x - 3$$

$$x = -(8) / (2(-2)) = 2$$

$$y = -2(2)^2 + 8(2) - 3 = 5$$

$$\text{Axis: } x = 2 \quad \text{Vertex: } (2, 5) \text{ (maximum)}$$

$$13. y = x^2 + 3x - 10$$

$$x = -(3) / (2(1)) = -3/2$$

$$y = 1(-3/2)^2 + 3(-3/2) - 10 = -49/4$$

$$\text{Axis: } x = -3/2 \quad \text{Vertex: } (-3/2, -49/4)$$

$$14. y = x^2 + 2x + 7$$

$$x = -(2) / (2(1)) = -1$$

$$y = 1(-1)^2 + 2(-1) + 7 = 6$$

$$\text{Axis: } x = -1 \quad \text{Vertex: } (-1, 6)$$

Answer Key - Continued

15. $y = 2x^2 - 10x + 8$
 $x = -(-10) / (2(2)) = 5/2$
 $y = 2(5/2)^2 - 10(5/2) + 8 = -9/2$
Axis: $x = 5/2$ Vertex: $(5/2, -9/2)$

16. $y = x^2 - 12x + 36$
 $x = -(-12) / (2(1)) = 6$
 $y = 1(6)^2 - 12(6) + 36 = 0$
Axis: $x = 6$ Vertex: $(6, 0)$ (on the x-axis)

17. $y = -3x^2 + 6x + 2$
 $x = -(6) / (2(-3)) = 1$
 $y = -3(1)^2 + 6(1) + 2 = 5$
Axis: $x = 1$ Vertex: $(1, 5)$ (maximum)

18. $y = x^2 - 7x + 12$
 $x = -(-7) / (2(1)) = 7/2$
 $y = 1(7/2)^2 - 7(7/2) + 12 = -1/4$
Axis: $x = 7/2$ Vertex: $(7/2, -1/4)$

19. $y = 5x^2 + 20x + 11$
 $x = -(20) / (2(5)) = -2$
 $y = 5(-2)^2 + 20(-2) + 11 = -9$
Axis: $x = -2$ Vertex: $(-2, -9)$

20. $y = x^2 + x - 6$
 $x = -(1) / (2(1)) = -1/2$
 $y = 1(-1/2)^2 + 1(-1/2) - 6 = -25/4$
Axis: $x = -1/2$ Vertex: $(-1/2, -25/4)$

Quick check: If $a > 0$, the vertex is a minimum. If $a < 0$, the vertex is a maximum.

Free to print for classroom and home use. Created by Burke Tutoring In Fremont.