Finding Quadratic Roots with the Quadratic Formula

A step-by-step guide with a worked example

For any quadratic in the form $ax^2 + bx + c = 0$

$$x = (-b \pm \sqrt{(b^2 - 4ac)}) / 2a$$

This works on every quadratic, even the ones that don't factor nicely. That's usually when you reach for it.

When to Reach for the Formula

Try factoring first; it's usually faster when it works. The formula is the fallback for everything else: when you can't find two numbers that multiply and add to the right values, or when the roots involve a square root that isn't a whole number. It always works, whether or not the quadratic factors nicely.

The Steps

Start by identifying a, b, and c.

Make sure the equation is set equal to zero first, then read the coefficients off in order.

Compute the discriminant, $b^2 - 4ac$.

This one number tells you what kind of roots to expect before you've finished the problem. If it's positive, you get two real roots. If it's exactly zero, you get one repeated root. If it's negative, there are no real roots at all.

Plug into the formula and simplify the square root.

If the number under the root has a perfect-square factor, pull it out — √40 becomes 2√10, for instance. Always simplify before rounding to a decimal.

Split the ± into two separate roots, and check your work.

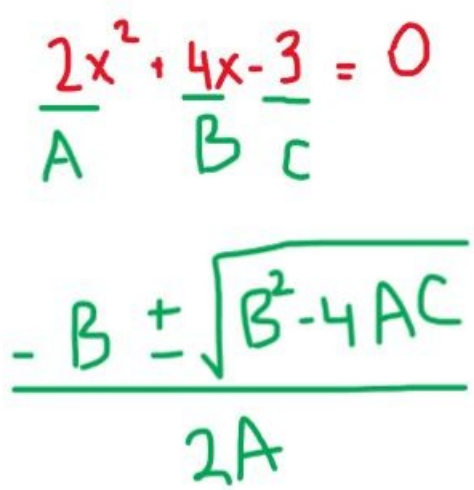
Write out both answers separately, then plug each back into the original equation. If it doesn't come out to zero, something went wrong upstream.

Worked Example

$$2x^2 + 4x - 3 = 0$$

Here's the same problem worked by hand, the way I actually do it with students.

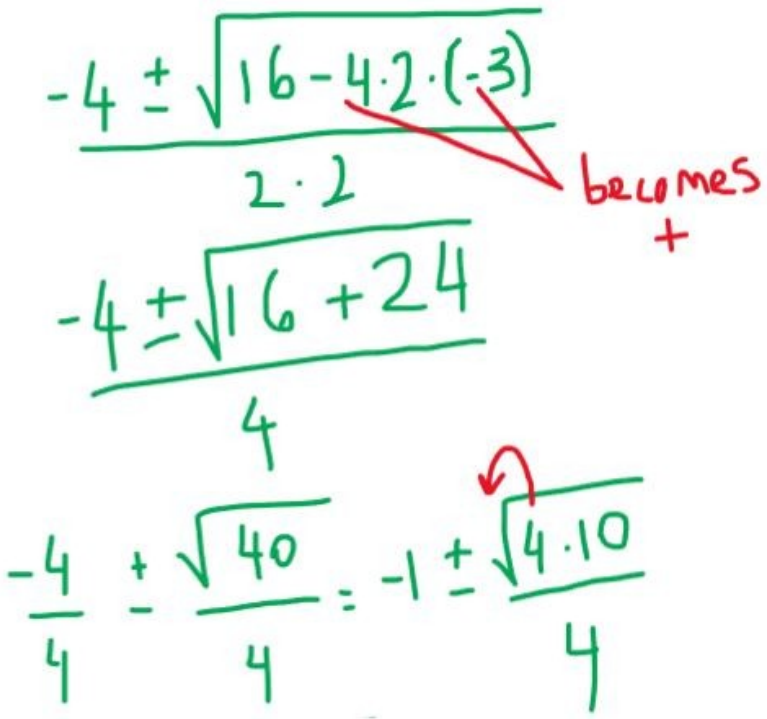
First, label a, b, and c, and write out the formula with those values in mind.



a = 2, b = 4, c = −3 labeled directly under the equation, with the formula written out underneath.

Substitute, and watch the sign when you multiply.

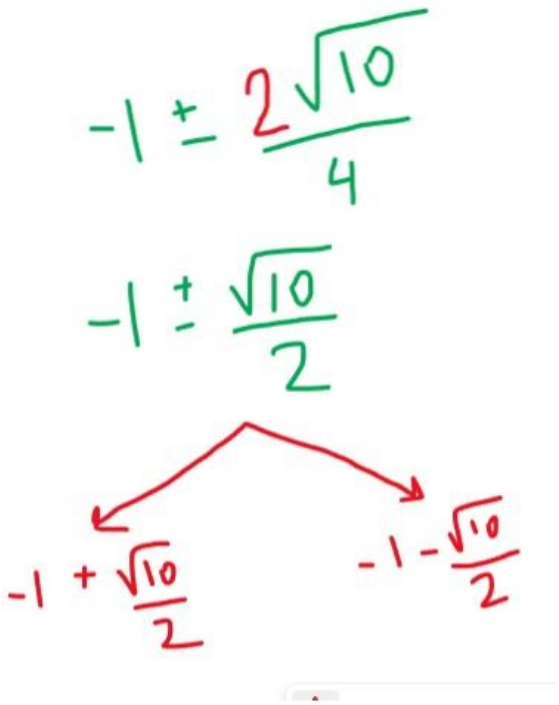
The most common slip in this whole process happens right here: −4 · 2 · (−3) involves two negatives, so the product comes out positive. It's worth pausing on that step specifically.



16 − 4(2)(−3) becomes 16 + 24, since the two negatives cancel — that's the step marked in red.

40 is positive, so this equation has two real roots. It isn't a perfect square, so the answer will keep a square root in it rather than reduce to whole numbers.

Simplify the radical, then split into the two roots.



2√10 over 4 reduces to √10 over 2 once the common factor of 2 divides out, then the ± splits into two separate answers.

x = −1 + √10⁄2, which is about 0.581
x = −1 − √10⁄2, which is about −2.581

Check the work.

Plugging x ≈ 0.581 back into 2x² + 4x − 3 gives roughly 0.675 + 2.324 − 3, which comes out to zero within rounding. Both roots hold up.

A Few Things Students Get Wrong Here

The ± sign is easy to forget, and dropping it silently loses one of the two roots. Sign errors on −b are common too: if b is −4, then −b is +4, not −4. Another frequent slip is dividing only part of the numerator by 2a instead of every term above the line. And it's worth double-checking that the square root is fully simplified before calling the problem done — an unsimplified radical is usually a sign a step got skipped.

Practice Problems

Use the quadratic formula on each one. Show the discriminant, then give both roots — or state that there are none.

1. $x^2 + 5x + 6 = 0$

x = _____

2. $x^2 - 3x - 10 = 0$

x = _____

3. $x^2 + 2x - 1 = 0$

x = _____

4. $x^2 - 6x + 9 = 0$

x = _____

5. $2x^2 + 7x + 3 = 0$

x = _____

6. $x^2 + 4x + 1 = 0$

x = _____

7. $x^2 - 2x + 5 = 0$

x = _____

8. $3x^2 - 5x - 2 = 0$

x = _____

9. $x^2 - 7 = 0$

x = _____

10. $2x^2 - 4x - 3 = 0$

x = _____

11. $x^2 + 6x + 4 = 0$

x = _____

12. $4x^2 + 4x + 1 = 0$

x = _____

13. $x^2 - 7x + 12 = 0$

x = _____

14. $2x^2 + 3x + 4 = 0$

x = _____

15. $x^2 + x - 3 = 0$

x = _____

16. $5x^2 - 6x + 1 = 0$

x = _____

17. $x^2 - 8x + 11 = 0$

x = _____

18. $3x^2 + 2x - 1 = 0$

x = _____

19. $x^2 - 12 = 0$

x = _____

20. $2x^2 - 6x + 1 = 0$

x = _____

Answer Key

Each one shows the coefficients, the discriminant, and the simplification step — so a student can find exactly where their work diverged.

1. $x^2 + 5x + 6 = 0$

$a = 1,\ b = 5,\ c = 6$

$b^2 - 4ac = (5)^2 - 4(1)(6) = 25 - (24) = 1$

$\sqrt{1} = 1$, so $x = (-5 \pm 1) / 2$

Answer: $x = -2,\ x = -3$

2. $x^2 - 3x - 10 = 0$

$a = 1,\ b = -3,\ c = -10$

$b^2 - 4ac = (-3)^2 - 4(1)(-10) = 9 - (-40) = 49$

$\sqrt{49} = 7$, so $x = (3 \pm 7) / 2$

Answer: $x = 5,\ x = -2$

3. $x^2 + 2x - 1 = 0$

$a = 1,\ b = 2,\ c = -1$

$b^2 - 4ac = (2)^2 - 4(1)(-1) = 4 - (-4) = 8$

$\sqrt{8}$ simplifies to $2\sqrt{2}$, so $x = (-2 \pm 2\sqrt{2}) / 2$

Answer: $x = -1 + \sqrt{2},\ x = -1 - \sqrt{2}$

4. $x^2 - 6x + 9 = 0$

$a = 1,\ b = -6,\ c = 9$

$b^2 - 4ac = (-6)^2 - 4(1)(9) = 36 - (36) = 0$

The discriminant is 0, so the ± contributes nothing and there's a single repeated root: $x = 6 / 2$

Answer: $x = 3$ **(repeated root)**

5. $2x^2 + 7x + 3 = 0$

$a = 2,\ b = 7,\ c = 3$

$b^2 - 4ac = (7)^2 - 4(2)(3) = 49 - (24) = 25$

$\sqrt{25} = 5$, so $x = (-7 \pm 5) / 4$

Answer: $x = -1/2,\ x = -3$

6. $x^2 + 4x + 1 = 0$

$a = 1,\ b = 4,\ c = 1$

$b^2 - 4ac = (4)^2 - 4(1)(1) = 16 - (4) = 12$

$\sqrt{12}$ simplifies to $2\sqrt{3}$, so $x = (-4 \pm 2\sqrt{3}) / 2$

Answer: $x = -2 + \sqrt{3},\ x = -2 - \sqrt{3}$

7. $x^2 - 2x + 5 = 0$

$a = 1,\ b = -2,\ c = 5$

$b^2 - 4ac = (-2)^2 - 4(1)(5) = 4 - (20) = -16$

The discriminant is −16. A negative discriminant means the parabola never crosses the x-axis.

Answer: **No real roots**

8. $3x^2 - 5x - 2 = 0$

a = 3, b = −5, c = −2
$b^2 - 4ac = (-5)^2 - 4(3)(-2) = 25 - (-24) = 49$
√49 = 7, so x = (5 ± 7) / 6
Answer: $x = 2,\ x = -1/3$

9. $x^2 - 7 = 0$
a = 1, b = 0, c = −7
$b^2 - 4ac = (0)^2 - 4(1)(-7) = 0 - (-28) = 28$
√28 simplifies to 2√7, so x = (0 ± 2√7) / 2
Answer: $x = \sqrt{7},\ x = -\sqrt{7}$

10. $2x^2 - 4x - 3 = 0$
a = 2, b = −4, c = −3
$b^2 - 4ac = (-4)^2 - 4(2)(-3) = 16 - (-24) = 40$
√40 simplifies to 2√10, so x = (4 ± 2√10) / 4
Answer: $x = (2 + \sqrt{10})/2,\ x = (2 - \sqrt{10})/2$

11. $x^2 + 6x + 4 = 0$
a = 1, b = 6, c = 4
$b^2 - 4ac = (6)^2 - 4(1)(4) = 36 - (16) = 20$
√20 simplifies to 2√5, so x = (−6 ± 2√5) / 2
Answer: $x = -3 + \sqrt{5},\ x = -3 - \sqrt{5}$

12. $4x^2 + 4x + 1 = 0$
a = 4, b = 4, c = 1
$b^2 - 4ac = (4)^2 - 4(4)(1) = 16 - (16) = 0$
The discriminant is 0, so the ± contributes nothing and there's a single repeated root: x = −4 / 8
Answer: $x = -1/2$ **(repeated root)**

13. $x^2 - 7x + 12 = 0$
a = 1, b = −7, c = 12
$b^2 - 4ac = (-7)^2 - 4(1)(12) = 49 - (48) = 1$
√1 = 1, so x = (7 ± 1) / 2
Answer: $x = 4,\ x = 3$

14. $2x^2 + 3x + 4 = 0$
a = 2, b = 3, c = 4
$b^2 - 4ac = (3)^2 - 4(2)(4) = 9 - (32) = -23$
The discriminant is −23. A negative discriminant means the parabola never crosses the x-axis.
Answer: **No real roots**

15. $x^2 + x - 3 = 0$
a = 1, b = 1, c = −3
$b^2 - 4ac = (1)^2 - 4(1)(-3) = 1 - (-12) = 13$
√13 has no perfect-square factor, so it stays as is, so x = (−1 ± √13) / 2
Answer: $x = (-1 + \sqrt{13})/2,\ x = (-1 - \sqrt{13})/2$

16. $5x^2 - 6x + 1 = 0$

a = 5, b = −6, c = 1
$b^2 - 4ac = (-6)^2 - 4(5)(1) = 36 - (20) = 16$
√16 = 4, so x = (6 ± 4) ⁄ 10
Answer: $x = 1,\ x = 1/5$

17. $x^2 - 8x + 11 = 0$
a = 1, b = −8, c = 11
$b^2 - 4ac = (-8)^2 - 4(1)(11) = 64 - (44) = 20$
√20 simplifies to 2√5, so x = (8 ± 2√5) ⁄ 2
Answer: $x = 4 + \sqrt{5},\ x = 4 - \sqrt{5}$

18. $3x^2 + 2x - 1 = 0$
a = 3, b = 2, c = −1
$b^2 - 4ac = (2)^2 - 4(3)(-1) = 4 - (-12) = 16$
√16 = 4, so x = (−2 ± 4) ⁄ 6
Answer: $x = 1/3,\ x = -1$

19. $x^2 - 12 = 0$
a = 1, b = 0, c = −12
$b^2 - 4ac = (0)^2 - 4(1)(-12) = 0 - (-48) = 48$
√48 simplifies to 4√3, so x = (0 ± 4√3) ⁄ 2
Answer: $x = 2\sqrt{3},\ x = -2\sqrt{3}$

20. $2x^2 - 6x + 1 = 0$
a = 2, b = −6, c = 1
$b^2 - 4ac = (-6)^2 - 4(2)(1) = 36 - (8) = 28$
√28 simplifies to 2√7, so x = (6 ± 2√7) ⁄ 4
Answer: $x = (3 + \sqrt{7})/2,\ x = (3 - \sqrt{7})/2$
