Solving Quadratics by Completing the Square

A free completing the square worksheet, with two worked examples, the slip that costs the most marks, and a full answer key.

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Factoring is quicker when it works. It stops working the moment the roots stop being whole numbers, and most quadratics you meet after Algebra I are like that. Completing the square never stalls. It also hands you the vertex on the way past, which factoring does not.

The whole method rests on one idea: you can add anything to an expression as long as you take it straight back off. Everything below is that idea plus bookkeeping.

THE METHOD IN FOUR STEPS

1. Keep the x^2 and x terms together. If a number sits in front of x^2, factor it out of those two terms only.
2. Halve the x coefficient, then square it.
3. Add that number and subtract it again in the same line.
4. Fold the first three terms into a square and tidy up what is left.

Worked Example: $y = x^2 - 6x + 2$

Half of -6 is -3, and $(-3)^2$ is 9. Add 9, subtract 9, and the first three terms collapse into $(x - 3)^2$. The two loose constants, -9 and $+2$, combine to -7.

Worked example: $y = x^2 - 6x + 2$

$$y = x^2 - 6x + 2$$

Half of −6 is −3, and $(-3)^2$ is 9

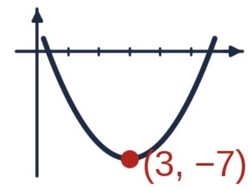
$$y = \underbrace{x^2 - 6x + 9}_{} - 9 + 2$$

These three terms are $(x - 3)^2$

The leftovers: −9 + 2 = −7

$$\boxed{y = (x-3)^2 - 7}$$ **Vertex (3, −7)**

Same curve, from the vertex



Add 9 and subtract 9 in the same line. Nothing changes, but the square appears.

When a Number Sits in Front of x^2

Factor it out of the x terms first, then complete the square inside the bracket. The one thing students forget: whatever you subtract inside gets multiplied by that number on the way out.

Worked example: $y = 3x^2 + 12x + 5$

$$y = 3(x^2 + 4x) + 5$$

Factor the 3 out of the x-terms only — leave the 5 alone

$$y = 3(\underbrace{x^2 + 4x + 4}_{} - 4) + 5$$

Half of 4 is 2, and 2^2 is 4, so the bracket holds $(x + 2)^2$

$$y = 3(x + 2)^2 - 12 + 5$$

The 3 multiplies the −4 on its way out: $3 \times (-4) = -12$

$$\boxed{y = 3(x + 2)^2 - 7}$$ **Vertex (−2, −7)**

✓ Check: expand it back out and you land on $3x^2 + 12x + 5$

The −4 inside the bracket leaves as −12, because the 3 multiplies it too.

The Slip Worth Knowing About

Almost every wrong answer on this topic comes from the same place. The 25 gets added, the square gets written, and the 25 never comes back off. The equation quietly changes into a different parabola.

One step, two endings: $y = x^2 + 10x + 3$

Half of 10 is 5, and 5^2 is 25 — everybody gets this far

✗ Then this happens	✓ What should happen
$y = (x^2 + 10x + 25) + 3$	$y = (x^2 + 10x + 25) - 25 + 3$
25 was added here and never taken back off	Added, then taken straight back off — nothing changes
$\cancel{y = (x + 5)^2 + 3}$	$\boxed{y = (x + 5)^2 - 22}$
~~Vertex (−5, 3)~~	**Vertex (−5, −22)**

Check with $x = 0$: the original gives 3 · the left column gives 28 · the right column gives 3

Red is the version that gets handed in. Green is the same first move, finished properly.

> **BALANCE THE LINE**
>
> If you add a number to one side of an equation and nothing else changes, the equation is no longer true. Either subtract it again in the same line, or add it to both sides. Pick one habit and keep it.

Solving vs. Rewriting

Two jobs, one method, slightly different endings.

Solving. Move the constant across, complete the square, then take the square root of both sides. The ± is not optional. $x - 3 = \pm 2$ gives two answers, and dropping the negative one loses half the marks.

Rewriting in vertex form. Keep everything on one side and stop at $y = a(x - h)^2 + k$. The vertex is (h, k), sign flipped. There is more on that in the vertex form and intercept form worksheet, and in the vertex and axis of symmetry worksheet if you'd rather get there straight from standard form. If the quadratic happens to factor cleanly, solving by factoring is faster, and the quadratic formula is really just this method run once on the general case — the discriminant tells you how many roots to expect before you start. Khan Academy's completing the square article and the OpenStax Algebra and Trigonometry chapter both cover the same ground if you want a second explanation.

Practice Problems: Completing the Square Worksheet

Part A is straight practice with a leading coefficient of 1. Part B adds a number in front of x^2. Show your work in the space provided.

Part A — solve by completing the square

1. $x^2 + 6x + 5 = 0$
2. $x^2 - 4x - 12 = 0$
3. $x^2 + 2x - 8 = 0$
4. $x^2 - 8x + 7 = 0$
5. $x^2 + 10x + 9 = 0$
6. $x^2 - 6x + 4 = 0$
7. $x^2 + 4x - 6 = 0$
8. $x^2 - 2x - 5 = 0$
9. $x^2 + 5x + 6 = 0$
10. $x^2 - 3x - 10 = 0$

Part B — problems 11–15: solve. Problems 16–20: write in vertex form and give the vertex.

1. $2x^2 + 8x + 6 = 0$

2. $3x^2 - 12x + 9 = 0$

3. $2x^2 - 4x - 3 = 0$

4. $4x^2 + 8x - 5 = 0$

5. $2x^2 + 6x + 1 = 0$

6. $y = 2x^2 + 8x + 3$

7. $y = 3x^2 - 6x + 1$

8. $y = x^2 + 7x + 10$

9. $y = -x^2 + 4x - 1$

0. $y = 5x^2 - 20x + 13$

Completing the Square Worksheet: Answer Key

Part A

1. $x^2 + 6x + 5 = 0$

Half of 6 is 3, squared is 9. $(x + 3)^2 - 9 + 5 = 0$, so $(x + 3)^2 = 4$ and $x + 3 = \pm 2$.

$x = -1$ and $x = -5$

2. $x^2 - 4x - 12 = 0$

$(x - 2)^2 - 4 - 12 = 0$, so $(x - 2)^2 = 16$ and $x - 2 = \pm 4$.

$x = 6$ and $x = -2$

3. $x^2 + 2x - 8 = 0$

$(x + 1)^2 - 1 - 8 = 0$, so $(x + 1)^2 = 9$ and $x + 1 = \pm 3$.

$x = 2$ and $x = -4$

4. $x^2 - 8x + 7 = 0$

$(x - 4)^2 - 16 + 7 = 0$, so $(x - 4)^2 = 9$ and $x - 4 = \pm 3$.

$x = 7$ and $x = 1$

5. $x^2 + 10x + 9 = 0$

$(x + 5)^2 - 25 + 9 = 0$, so $(x + 5)^2 = 16$ and $x + 5 = \pm 4$.

$x = -1$ and $x = -9$

6. $x^2 - 6x + 4 = 0$

$(x - 3)^2 - 9 + 4 = 0$, so $(x - 3)^2 = 5$. This one does not factor, which is the point.

$x = 3 \pm \sqrt{5}$

7. $x^2 + 4x - 6 = 0$

$(x + 2)^2 - 4 - 6 = 0$, so $(x + 2)^2 = 10$.

$x = -2 \pm \sqrt{10}$

8. $x^2 - 2x - 5 = 0$

$(x - 1)^2 - 1 - 5 = 0$, so $(x - 1)^2 = 6$.

$x = 1 \pm \sqrt{6}$

9. $x^2 + 5x + 6 = 0$

Half of 5 is 5/2, squared is 25/4. $(x + 5/2)^2 - 25/4 + 6 = 0$, so $(x + 5/2)^2 = 1/4$ and $x + 5/2 = \pm 1/2$.

$x = -2$ and $x = -3$

10. $x^2 - 3x - 10 = 0$

$(x - 3/2)^2 - 9/4 - 10 = 0$, so $(x - 3/2)^2 = 49/4$ and $x - 3/2 = \pm 7/2$.

$x = 5$ and $x = -2$

Part B

11. $2x^2 + 8x + 6 = 0$

Divide through by 2 first: $x^2 + 4x + 3 = 0$. Then $(x + 2)^2 = 1$.

$x = -1$ and $x = -3$

12. $3x^2 - 12x + 9 = 0$

Divide by 3: $x^2 - 4x + 3 = 0$. Then $(x - 2)^2 = 1$.

$x = 1$ and $x = 3$

13. $2x^2 - 4x - 3 = 0$

Divide by 2: $x^2 - 2x = 3/2$. Then $(x - 1)^2 = 5/2$.

$x = 1 \pm \sqrt{10} / 2$

14. $4x^2 + 8x - 5 = 0$

Divide by 4: $x^2 + 2x = 5/4$. Then $(x + 1)^2 = 9/4$ and $x + 1 = \pm 3/2$.

$x = 1/2$ and $x = -5/2$

15. $2x^2 + 6x + 1 = 0$

Divide by 2: $x^2 + 3x = -1/2$. Then $(x + 3/2)^2 = 7/4$.

$x = (-3 \pm \sqrt{7}) / 2$

16. $y = 2x^2 + 8x + 3$

$2(x^2 + 4x) + 3 = 2(x^2 + 4x + 4 - 4) + 3 = 2(x + 2)^2 - 8 + 3$.

$y = 2(x + 2)^2 - 5$ · Vertex $(-2, -5)$

17. $y = 3x^2 - 6x + 1$

$3(x^2 - 2x) + 1 = 3(x^2 - 2x + 1 - 1) + 1 = 3(x - 1)^2 - 3 + 1$.

$y = 3(x - 1)^2 - 2$ · Vertex $(1, -2)$

18. $y = x^2 + 7x + 10$

Half of 7 is 7/2, squared is 49/4. $(x + 7/2)^2 - 49/4 + 10$.

$y = (x + 7/2)^2 - 9/4$ · Vertex $(-7/2, -9/4)$

19. $y = -x^2 + 4x - 1$

Factor out −1: $-(x^2 - 4x) - 1 = -(x^2 - 4x + 4 - 4) - 1 = -(x - 2)^2 + 4 - 1$.

$y = -(x - 2)^2 + 3$ · Vertex $(2, 3)$

20. $y = 5x^2 - 20x + 13$

$5(x^2 - 4x) + 13 = 5(x^2 - 4x + 4 - 4) + 13 = 5(x - 2)^2 - 20 + 13$.

$y = 5(x - 2)^2 - 7$ · Vertex $(2, -7)$

Keep Practicing

Once the vertex is second nature, graphing quadratic functions is the natural next step, and working backward with the parabola equation from a graph worksheet checks whether it actually stuck.